Accuracy of Evapotranspiration Rates Determined by the Water-Budget Method, Gila River Flood Plain, Southeastern Arizona

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GILA RIVER PHREATOPHYTE PROJECT

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SYMBOLS

Symbol	Definition
a	constant defined by equation 23
A	area of reach, surface area assigned to a sample point, wetted cross-sectional area of Gila River channel, or subscript denoting "after clearing"
b	constant defined by equation 24 or subscript denoting a bias type of error
B	subscript denoting "basin fill" or "before clearing"
c	constant defined by equation 25
C	subscript denoting "capillary zone of soil profile"
ΔC	change in Gila River channel storage
d	subscript denoting a given budget period day
D	total number of days in budget period
e_1	temporal variability of Gila River wetted cross-sectional area
e_2	spatial variability of Gila River wetted cross-sectional area
e_3	error in Gila River wetted cross-sectional area due to error in discharge used to compute area
e_q	error in Gila River instantaneous (or average daily) discharge
E_q	average standard error of mean daily discharge of the Gila River for budget period
E_x	bias error, adjusted sampling error, or total measurement error of water-budget component or parameter indicated by subscript x
ET, $\overline{ET}$	evapotranspiration, average evapotranspiration
$\Delta \overline{ET}$	average change in evapotranspiration
G_B	basin-fill inflow
G_I, G_O	downvalley ground-water inflow and outflow
Δh, $\Delta \overline{h}$	ground-water level change, average ground-water level change
i	downvalley ground-water slope
I	subscript denoting "inflow" or "intermediate zone of soil profile"
j	sample point—precipitation gage, soil-moisture access hole, or ground-water well
k	total number of observations (budget periods)
L	length of Gila River channel
m	number of sample points $\leq n$
M_{zjt}	soil-moisture content in zone z of hole j at the end of budget period t
ΔM_{zjt}, $\Delta \overline{M}$	change in soil-moisture content in zone z of hole j at the end of budget period t, and average change in soil-moisture content
n	total number of sample points in reach
n'	largest even value $\leq n$
N	number of discharge measurements made during the budget period
O	subscript denoting "outflow"
p	total number of permutations
P, $\overline{P}$	precipitation and average precipitation
q	instantaneous or average daily discharge
Q_I, Q_O	Gila River inflow and outflow during a budget period
Q_T	tributary inflow
r	approximate correlation coefficient
R_{jt}	departure of observation at sample point j from average of observations at n sample points for budget period t
$\overline{R}_j$	average departure of k observations at sample point j
$\overline{R}_{mv}$	mean departure of observations at m sample points from observations at n sample points for permutation v

s	subscript denoting a sampling type of error
s_j	standard deviation of R_j
$s_{\overline{ET}}$	standard deviation of average ET
$s_{\Delta\overline{ET}}$	standard deviation of average change in ET
S	subscript denoting "soil zone of soil profile"
S_{mv}	standard deviation of R_{mv}
$\overline{S}_m$	average standard deviation of all R_{mv} computed from p permutations (missing-data error)
S'	apparent specific yield
t	subscript denoting a given budget period
T	transmissivity or subscript denoting "tributary"
TC, TI, TS	subscripts denoting "capillary," "intermediate," and "soil" zones of terrace soil profile
v	subscript denoting a given permutation of the sample points
W	width of saturated alluvium
z	subscript denoting a given soil-moisture zone
ϵ	expectation
ξ	residual error in ratio of $\overline{S}^2_m$ to $\overline{S}^2_{2m}$
ρ_{j1j2}	estimate of population correlation coefficient between sample points j_1 and j_2
∇	unadjusted sampling error for a complete set of sample points

METRIC-ENGLISH EQUIVALENTS

Metric unit		English equivalent
Length		
millimetre (mm)	= 0.03937	inch (in)
metre (m)	= 3.28	feet (ft)
kilometre (km)	= .62	mile (mi)
Area		
square metre (m^2)	= 10.76	square feet (ft^2)
square kilometre (km^2)	= .386	square mile (mi^2)
hectare (ha)	= 2.47	acres
Volume		
cubic centimetre (cm^3)	= 0.061	cubic inch (in^3)
litre (l)	= 61.03	cubic inches
cubic metre (m^3)	= 35.31	cubic feet (ft^3)
cubic metre	= .00081	acre-foot (acre-ft)
cubic hectometre (hm^3)	= 810.7	acre-feet
litre	= 2.113	pints (pt)
litre	= 1.06	quarts (qt)
litre	= .26	gallon (gal)
cubic metre	= .00026	million gallons (Mgal or 10^6 gal)
cubic metre	= 6.290	barrels (bbl) (1 bbl = 42 gal)
Weight		
gram (g)	= 0.035	ounce, avoirdupois (oz avdp)
gram	= .0022	pound, avoirdupois (lb avdp)
tonne (t)	= 1.1	tons, short (2,000 lb)
tonne	= .98	ton, long (2,240 lb)
Specific combinations		
kilogram per square centimetre (kg/cm^2)	= 0.96	atmosphere (atm)
kilogram per square centimetre	= .98	bar (0.9869 atm)
cubic metre per second (m^3/s)	= 35.3	cubic feet per second (ft^3/s)

Metric unit		English equivalent
Specific combinations—Continued		
litre per second (l/s)	= .0353	cubic foot per second
cubic metre per second per square kilometre [(m^3/s)/km^2]	= 91.47	cubic feet per second per square mile [(ft^3/s)/mi^2]
metre per day (m/d)	= 3.28	feet per day (hydraulic conductivity) (ft/d)
metre per kilometre (m/km)	= 5.28	feet per mile (ft/mi)
kilometre per hour (km/h)	= .9113	foot per second (ft/s)
metre per second (m/s)	= 3.28	feet per second
metre squared per day (m^2/d)	= 10.764	feet squared per day (ft^2/d) (transmissivity)
cubic metre per second (m^3/s)	= 22.826	million gallons per day (Mgal/d)
cubic metre per minute (m^3/min)	= 264.2	gallons per minute (gal/min)
litre per second (l/s)	= 15.85	gallons per minute
litre per second per metre [(l/s)/m]	= 4.83	gallons per minute per foot [(gal/min)/ft]
kilometre per hour (km/h)	= .62	mile per hour (mi/h)
metre per second (m/s)	= 2.237	miles per hour
gram per cubic centimetre (g/cm^3)	= 62.43	pounds per cubic foot (lb/ft^3)
gram per square centimetre (g/cm^2)	= 2.048	pounds per square foot (lb/ft^2)
gram per square centimetre	= .0142	pound per square inch (lb/in^2)
Temperature		
degree Celsius (°C)	= 1.8	degrees Fahrenheit (°F)
degrees Celsius (temperature)	= [(1.8×°C)+32]	degrees Fahrenheit

GILA RIVER PHREATOPHYTE PROJECT

ACCURACY OF EVAPOTRANSPIRATION RATES DETERMINED BY THE WATER-BUDGET METHOD, GILA RIVER FLOOD PLAIN, SOUTHEASTERN ARIZONA

By RONALD L. HANSON and DAVID R. DAWDY

ABSTRACT

Evapotranspiration by phreatophytes (primarily saltcedar) was determined by the water-budget method for 5,500 acres (2,230 ha) of the Gila River flood plain in southeastern Arizona. The water budget consists of 12 components including surface and subsurface flow through the study area, precipitation on the area, and soil-moisture changes in the unsaturated soil profile.

Nine years (1963–71) of hydrologic data were collected on four reaches within the area. These data provided over 400 measurements of evapotranspiration for two- or three-week periods. Midway through the study the vegetation was removed from the flood plain. The evapotranspiration measurements are therefore defined for both natural vegetative cover and essentially bare-ground conditions.

This report shows how each component of the water budget was evaluated, demonstrates the significance of each component in relation to the total evapotranspiration, and describes the methods used to evaluate the relative accuracy of each component.

The two most significant components of the water budget are, generally, the Gila River inflow and outflow. One of the least significant is tributary inflow, which occurred only 4 percent of the time during the 9-year study. Soil-moisture change is highly significant during periods of low streamflow and is one of the more difficult components to measure. The ground-water flow components are the least variable in the water budget, fluctuating only in response to seasonal changes in the downvalley ground-water slope.

The total measurement error of each component consists primarily of a sampling error which is dependent on the number of observation points used to measure the component. This error is time variant, reflecting both the variability in repetitive measurements and the error due to missing data. Included in the total measurement error is a bias error which gives a constant overestimate or underestimate of the component. Only the ground-water flow components introduce a measurable bias error, but the direction of this error is unknown and its magnitude in relation to evapotranspiration is relatively insignificant.

The total measurement error in evapotranspiration is not related to the magnitude of evapotranspiration but rather to the total volume of water moving through the area. Thus, the minimum errors occur during the midsummer months of maximum evapotranspiration when streamflow is low and precipitation is negligible.

Evapotranspiration rates computed for reach 1 indicate that phreatophyte clearing reduced summer rates by nearly 45 percent. The average computed measurement errors in summer evapotranspiration rates, before and after clearing, are ±59 percent and ±113 percent, respectively, and the average measurement error in the change in summer evapotranspiration as a result of clearing is nearly ±200 percent. These large computed measurement errors are shown to overestimate substantially the true measurement variable in evapotranspiration. The computed errors do give, however, a good indication of the relative significance of each evapotranspiration value and provide a means of selecting those values which should be used in computing average evapotranspiration rates. Furthermore, the results of this error analysis show that reliable estimates of summer evapotranspiration can be determined and that a significant difference in summer evapotranspiration could be detected as a result of clearing phreatophytes from the flood plain.

INTRODUCTION

The determination of *ET* (evapotranspiration) by phreatophytes from a flood plain by the water-budget method requires that all significant movement of liquid water into and out of the area be measured.

Components of the water budget include surface and subsurface flow through the area, precipitation on the area, and soil-moisture content changes in the unsaturated zone of the area. It has been the general opinion of most researchers that the measurement errors associated with these components are too large to provide reliable estimates of *ET*—particularly when an estimate of water salvage as a result of phreatophyte removal is desired. To date (1976) few studies have been conducted which evaluated *ET* from a large area (Gatewood and others, 1950; Turner and Skibitzke, 1952; Bowie and Kam, 1968), and little is known about the accuracy of these evaluations. In October 1962 a nine-year water-budget study began on 5,500 acres (2,230 ha) of the Gila River flood plain in southeastern Arizona. The primary objective of this study was to evaluate seasonal *ET* rates of phreatophytes from the area and water salvage following removal of the phreatophytes (Culler and others, 1970).

The study was designed and instrumented to measure independently each component of the water budget. The purposes of this report are to show how each component of the water budget was derived, to describe the method used in estimating the accuracy of each component, and to demonstrate the significance of each component and its error in the resulting *ET* values. The errors for most of the water-budget components are expressed in terms of the standard error of their measurement. For some components, however, the measurement error can only be approximated. The assumptions and criteria used to arrive at these approximations were defined to provide a resultant error which, in most instances, should exceed the expected standard error in the measurement of the component. The derived error of each *ET* value is therefore considered to be only a *relative* indicator of measurement variability in *ET*.

Two types of error were investigated in this analysis—the bias error and the sampling error. The bias error is a constant time-invariant error caused by consistent overestimates or underestimates of the true value of the component. When evaluating the average change in evapotranspiration ($\Delta\overline{ET}$)—such as occurs after clearing phreatophytes from the flood plain—this error cancels and is thus not included in the determination of the accuracy of $\Delta\overline{ET}$. However, when evaluating absolute values of *ET*, the bias error may, in some instances, be a significant part of the total error in *ET*.

The sampling error reflects the variability in the measurement of a water-budget component due to insufficient sampling of the component. This error decreases with an increase in the number of observations at the sample point or with an increase in the number of sample points, and it increases with an increase in the magnitude of the component. The sampling error is time dependent for those components in which the number of sampling points and the magnitude changes during the study period.

The total measurement error of each *ET* value was obtained from the sum of squares of the bias and sampling errors defined for each component. Because of independence of the components no covariance term exists in the computation of the total measurement error. Furthermore, this total error term is considered to be only a *relative* indicator of the measurement variability in *ET* and not an estimate of the expected standard error in *ET*.

This study was conducted under the general supervision of R. C. Culler, project chief of the Gila River Phreatophyte Project. Transformation of basic field data into a form acceptable for analysis was performed by R. M. Myrick and F. P. Kipple. The authors are indebted to the San Carlos Apache Indian Tribe and the Bureau of Indian Affairs for the use of their lands and facilities, respectively, to make this study.

DESCRIPTION OF STUDY AREA

The study area includes a 15-mile (24 km) length of the Gila River flood plain above San Carlos Reservoir in southeastern Arizona (fig. 1). The flood plain averages 1 mile (1.6 km) in width and has a gradual downvalley slope of about 1.5 ft per 1,000 ft. The water-bearing deposits in the flood plain consist of basin-fill deposits and alluvial deposits which overlie the basin fill. The basin-fill deposits are more than 1,000 ft (300 m) thick and consist of fine-grained material of low permeability. The alluvial deposits are as much as 60 ft (20 m) thick and consist of lenticular gravel, sand, and silt beds with a relatively high permeability. The alluvial deposits form the flood plain and lower terraces in the study area. The Gila River meanders across the alluvium in a channel averaging 110 ft (35 m) wide and 7 ft (2 m) deep. A detailed description of the geology of the study area is given by Weist (1971).

The depth to ground water ranges from about 5 ft (1½ m) near the river to more than 20 ft (6 m) near the outer boundaries of the flood plain. Wells that penetrate through the flood-plain alluvium into the underlying basin fill indicate that ground water in the basin fill flows vertically upward into the alluvium at a rate of about 0.3 ft (0.09 m) per year or 0.1 million gallons per day per acre (0.01 $m^3/s/ha$). Downvalley ground-water movement through the alluvium averages about 5.1 acre-ft per day (0.0063 hm^3/day) (Hanson, 1972). This downvalley flow is equivalent to 1.7 million gallons per day (180,000 m^3/s).

Gila River inflow to the study area is derived from 11,500 mi^2 (29,800 km^2) of drainage area. Most of the streamflow results from winter and late summer precipitation. The average discharge of the Gila River is 250 ft^3/s (7.1 m^3/s) but can range from no flow for a few days in the summer to several thousand cubic feet per second during the winter and summer storm periods.

Tributary inflow is derived from 225 mi^2 (583 km^2) of drainage area bordering the study area. Annual runoff from these tributaries is small and generally occurs only for short periods during the summer as a result of thunderstorms.

Precipitation occurs primarily in December and January from large frontal storms and in July, August, and September from short-duration high-intensity convective storms. The average annual precipitation at San Carlos Reservoir is 14 in. (360 mm), but annual totals have ranged from 4.0 in. (102 mm) to 25.8 in. (655 mm) during the period of record, 1882–1973.

Mean daily temperatures in the study area range from a minimum of 32°F (0°C) during the winter to a maximum of 100°F (38°C) in the summer. Pan evaporation at San Carlos Reservoir averages 97 in. (2,460 mm) per year, and evaporation from the reservoir is estimated at 70 in. (1,780 mm) per year (F. P. Kipple, written commun., 1975).

When the study began (October 1962) nearly 70 percent of the vegetation on the flood plain was saltcedar (*Tamarix chinensis*)[1], with the remaining vegetation consisting of mesquite (*Prosopis*), willow (*Salix*), cottonwood (*Populus*), seepwillow (*Baccharis*), and seepweed (*Suaeda*). Vegetation was removed from the flood plain in segments of several acres at different periods during 1967–71. Virtually all the study area was cleared of vegetation by March 1971.

THE WATER-BUDGET EQUATION

Twelve components are significant in defining *ET* from the water budget of the Gila River flood plain. An equation expressing these components is

$$ET = Q_I - Q_O + Q_T + \Delta C + \overline{P} + \Delta\overline{M}_S + \Delta\overline{M}_I + \Delta\overline{M}_C + G_B + G_I - G_O + \Delta\overline{M}_{TC}, \quad (1)$$

where

ET = evapotranspiration from the area,
Q_I = surface inflow of the Gila River,
Q_O = surface outflow of the Gila River,
Q_T = surface inflow from tributaries bordering the area,
ΔC = change in channel storage in the Gila River,
$\overline{P}$ = average precipitation on the area,
$\Delta\overline{M}_S$ = average change in moisture content in the unsaturated soil zone located immediately below the land surface,

[1]Also referred to as *Tamarix pentandra* and *Tamarix gallica*.

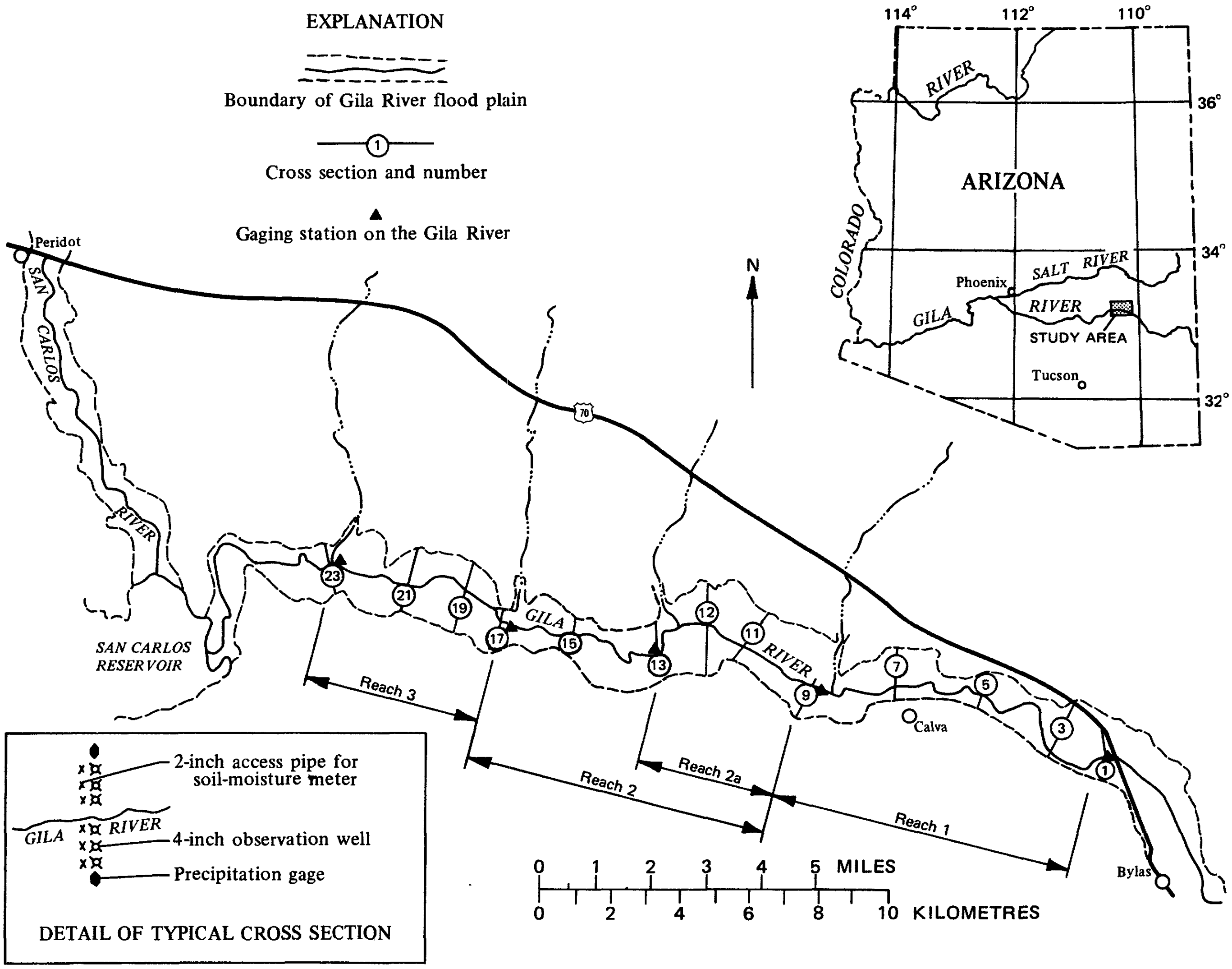

FIGURE 1.—Map showing study area and instrumentation location.

$\Delta \overline{M}_I$ = average change in moisture content in the unsaturated intermediate zone located between the overlying soil zone and the underlying capillary zone,

$\Delta \overline{M}_C$ = average change in moisture content in the capillary zone located below the intermediate zone and within the zone of water-table fluctuation,

G_B = ground-water inflow vertically upward into the alluvium from the underlying basin fill,

G_I = ground-water inflow downvalley through the saturated alluvium,

G_O = ground-water outflow downvalley through the saturated alluvium,

$\Delta \overline{M}_{TC}$ = average lateral ground-water movement through the capillary zone between the flood plain and the adjacent terrace area.

The basic data used to compute each component of the water budget were collected at the 13 cross sections shown in figure 1. Each cross section included three ground-water level observation wells equipped with recorders on each side of the Gila River, an access hole for measuring soil-moisture content adjacent to each well, and a nonrecording precipitation gage at both ends of each cross section (see inset showing detail of typical cross section in fig. 1). Recording precipitation gages were established at both ends of cross sections 1, 9, 17, and 23. Streamflow gaging stations were established at cross sections 1, 9, 13, 17, and 23 to define the Gila River inflow and outflow through reaches 1, 2, 2a, and 3 as shown in figure 1. Tributary inflow was measured at 16 continuous-recording gaging stations and 43 crest-stage gages (not shown in fig. 1) located along the perimeter of the study area. A more detailed discussion of the instrumentation is given by Culler and others (1970).

To solve for ET in equation 1 all basic field data were first transformed into terms described by the components in equation 1. Table 1 shows the water-budget components obtained from the basic field data in reach 1 and corresponding ET values for the 22 budget periods included in the 1964 water year (Oct. 1, 1963, to Sept. 30, 1964). The budget period includes either a two-week or a three-week period, depending on when field measurements of soil moisture were obtained. The end date shown in column 1 of table 1 refers to the last day of each budget period. The project day shown in column 2 refers to the ending day of the budget period referenced from the day the study began on October 1, 1962. Table 2 shows the water-budget components, the resulting ET, and their corresponding errors for the 21-day budget period 688 to 708 (Aug. 18 to Sept. 7, 1964). The following sections of this report describe how each ET component in table 2 was derived and discusses the methods used in deriving the sampling and bias errors associated with each component.

EVALUATION OF WATER-BUDGET COMPONENTS AND ERRORS

STREAMFLOW

The Gila River inflow (Q_I) and outflow (Q_O) were obtained by summing the computed daily discharges over the budget period at the upstream and downstream ends of the reach, respectively. For reach 1, Q_I was obtained from the computed daily discharges at cross section 1, and Q_O was obtained from the computed daily discharges at cross section 9 (fig. 1). The total volume of inflow during the 21-day period in table 2 is 1,051 acre-ft (1.296 hm^3), and the total volume of outflow is 1,107 acre-ft (1.365 hm^3). Table 3 lists the daily discharges used for obtaining the volume of inflow Q_I through cross section 1 during budget period 688–708. The difference between Q_I and Q_O is −56 acre-ft (−0.069 hm^3), indicating a net inflow to the river through the reach.

The accuracy of the computed volume of water passing a gaging station during a budget period is dependent on the measurement error in discharge and the accuracy of the stage-discharge relation defined for the station. The channel of the Gila River is subject to considerable scour and fill; thus, good definition of the stage-discharge relation requires frequent discharge measurements. Burkham and Dawdy (1970, figs. 11 and 12) developed curves of the relation between the standard error in a computed instantaneous discharge obtained from the rating curve and the frequency of discharge measurements for the stations at cross sections 1 and 9. The data used in their analysis was restricted to flows below a bankfull discharge of about 4,000 ft^3/s (100 m^3/s). Their error curves were developed on the assumption that the standard error of any given measured discharge is 4 percent as indicated by Carter and Anderson (1963, fig. 1), and they show that the standard error in a computed instantaneous discharge obtained from the stage-discharge relation is greater for the summer months (July through October) than for the winter and spring months (November through June). Burkham and Dawdy's error curves for the summer months at cross sections 1 and 9 were averaged to define the error curves shown in the semilog plot of figure 2 for measurement frequencies of one measurement every 3 days, every 5 days, and every 12 days. Because the curves in figure 2 were developed from only the summer data, their application to winter flows may give estimates of the standard error in a computed discharge which are too high. The frequency of discharge measurements for the Gila River during the study

TABLE 1.—*Evapotranspiration and water-budget components for 1964 water year, reach 1*

[All values in acre-ft for indicated days. Flood-plain area is 1,723 acres (697 ha)]

End date	Project day	Number of days	*ET*	Q_I	Q_O	ΔC	Q_T	$\bar{P}$	$\Delta \bar{M}_S$	$\Delta \bar{M}_I$	$\Delta \bar{M}_C$	G_B	G_I	G_O	$\Delta \bar{M}_{TC}$
1963															
Oct. 15	380	14	293	983	−960	28	1	0	7		137	41	87	−77	46
Oct. 29	394	14	582	11,960	−11,288	−42	21	96	−24		−128	41	87	−77	−64
Nov. 12	408	14	137	5,036	−5,022	−13		36	26		30	41	85	−77	−5
Nov. 26	422	14	209	4,107	−3,873	17		50	−23		−70	41	83	−77	−46
Dec. 10	436	14	525	3,491	−3,131	7		24	7		71	41	83	−77	9
Dec. 24	450	14	494	2,691	−2,316	18		0	10		−17	41	82	−76	61
1964															
Jan. 7	464	14	230	2,527	−2,269	−29		0	0		−18	41	83	−76	−29
Jan. 21	478	14	−604	4,203	−4,474	−5		0	−18		−166	41	82	−76	−191
Feb. 4	492	14	−40	4,322	−4,401	7		29	−2		6	41	82	−76	−48
Feb. 18	506	14	97	2,185	−2,241	33		0	9		17	41	83	−76	46
Mar. 3	520	14	9	1,015	−1,108	3		48	−20		17	41	84	−76	5
Mar. 17	534	14	195	958	−898	5		35	6		21	41	85	−76	18
Mar. 31	548	14	196	977	−928	0		26	−2		7	41	85	−76	66
Apr. 14	562	14	112	987	−890	0		75	−17		−51	41	85	−76	−42
May 4	582	20	269	1,166	−999	4	0	4	16		27	59	121	−107	−22
May 25	603	21	410	688	−600	8	0	0	23		127	62	128	−113	87
Jun. 15	624	21	372	173	−144	7	0	0	17		162	62	131	−113	77
Jul. 6	645	21	424	8	−3	0	0	5	15		145	62	132	−114	174
Jul. 27	666	21	1,056	7,614	−6,839	0	14	93	6		36	62	130	−115	55
Aug. 17	687	21	1,903	18,502	−17,068	98	176	272	−61		−86	62	128	−113	−7
Sep. 7	708	21	513	1,051	−1,107	23	252	174	8		82	62	124	−112	−44
Sep. 28	729	21	2,799	24,456	−21,821	−165	310	289	−73		−272	62	123	−113	3

TABLE 2.—*Evapotranspiration and water-budget components for the 21-day budget period 688–708 (August 18–September 7, 1964) in reach 1 and corresponding sampling and bias errors of each component*

[All figures in acre-ft per 21 days]

	ET	Q_I	Q_O	ΔC	Q_T	$\bar{P}$	$\Delta \bar{M}_S$	$\Delta \bar{M}_I$	$\Delta \bar{M}_C$	G_B	G_I	G_O	$\Delta \bar{M}_{TC}$
Component value	513	1,051	−1,107	23	252	174	8	--	82	62	124	−112	−44
Sampling error	334	107	121	12	252	16	20	--	42	0	0	0	138
Bias error	69	0	0	--	----	0	0	--	0	62	23	21	0
Total measurement error	$E_{ET}=(334^2+69^2)^{1/2}=\pm$ 341 acre-ft per 21 days												

TABLE 3.—*Daily discharges and errors in discharge at cross section 1 for budget period 688–708*

Date (1964)	q (ft³/s)	e_q[1] (dimensionless)	$(e_q \times q)^2$ (ft³/s)²
August			
18	47.0	0.133	39.1
19	29.0	.142	17.0
20	32.0	.140	20.2
21	55.0	.130	51.3
22	24.0	.146	12.2
23	18.0	.151	7.4
24	11.0	.160	3.1
25	9.2	.164	2.3
26	185.0	.108	395.6
27	20.0	.149	8.9
28	11.0	.160	3.1
29	11.0	.160	3.1
30	9.1	.164	2.2
31	9.6	.163	2.4
September			
1	10.0	.162	2.6
2	9.6	.163	2.4
3	9.2	.164	2.3
4	7.9	.166	1.7
5	7.2	.168	1.5
6	8.2	.166	1.8
7	7.0	.169	1.4
Totals	530.0		581.6

Q_I = 530.0 × 1.9835 = 1,051 acre-ft/21 days

E_q = $(581.6/21)^{1/2}$ = ±5.26 ft³/s = ±10.4 acre-ft/day

E_{Q_I} = 21 $((5.26)^2/4.2)^{1/2}$ = ±53.9 ft³/s/21 days = ±107 acre-ft/21 days

[1]From figure 2 (or equation 2).

period averaged about one every 5 days; thus, the 5-day curve in figure 2 was used in this analysis. The equation for this curve when $q \leq 4{,}000$ ft³/s (113 m³/s) is

$$e_q = 0.205 - 0.043 \log_{10} q, \quad (2)$$

where e_q is the error, expressed as a fraction of the instantaneous discharge, q, in cubic feet per second. For discharges above 4,000 ft³/s (113 m³/s), the error is assumed to increase linearly by the relation

$$e_q = -1.75 + 0.50 \log_{10} q. \quad (3)$$

Equation 3 assumes that the error in flows above bankfull stage increases from 5 percent of the discharge at q=4,000 ft³/s (113 m³/s) to 25 percent of the discharge at q=10,000 ft³/s (283 m³/s). Estimates of the error in discharge during overbank flooding in the Gila River as defined by equation 3 are believed to be high because most of the flow in the 4,000 to 10,000 ft³/s range is contained within the main channel where measurement errors are minimal. Equation 3 is not considered applicable to discharges above 10,000 ft³/s (283 m³/s).

Burkham and Dawdy (1970) showed that the standard error, E_Q, in the volume of flow passing a station during the budget period is

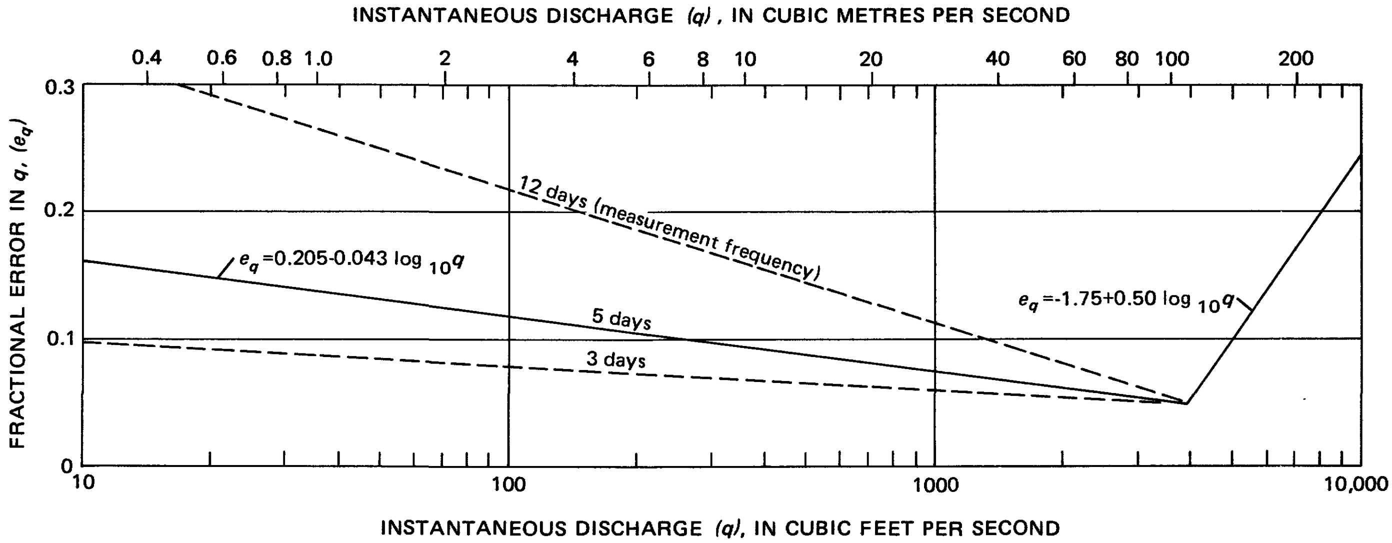

FIGURE 2.—Average relation between instantaneous discharge and the error in discharge expressed as a fraction of the discharge for summer (July through October) flows of the Gila River at cross sections 1 and 9.

$$E_Q=D\left(E_q^2/N\right)^{1/2}, \quad (4)$$

where E_q is the average standard error in the mean daily discharge for the budget period expressed as a fraction of the discharge, D is the number of days in the budget period, and N is the number of discharge measurements made during the budget period. E_q is obtained from the relation

$$E_q=\left[\sum_{d=1}^{D}\left(e_{q_d}\times q_d\right)^2/D\right]^{1/2}, \quad (5)$$

where q_d is the average discharge for day d and e_{q_d} is the fractional error in q_d as defined by equation 2 or 3. Only the discharge measurements provide an independent estimate of the daily discharge—all other daily discharges are computed from the stage-discharge relation and are therefore dependent estimates. Equation 4 considers this dependency by including the number of measurements, $N=D/5$, made during the budget period. The application of equation 2 or 3 to obtain E_q assumes that the error for a daily discharge is the same as the error for an instantaneous discharge. This assumption is invalid only during wide fluctuations in discharge which occur for a few days during the winter snowmelt runoff and the late summer thunderstorm periods.

Included in table 3 are the computations for determining the standard error of the volume of inflow through cross section 1, E_{Q_I}, during budget period 688–708. The fractional errors, e_q, shown for each daily discharge were obtained from figure 2 (or equation 2). Applying equation 5 gives the average error for the 21 daily discharges of $E_q=\pm 10.4$ acre-ft (± 0.0129 hm^3) per day. If one discharge measurement is made every 5 days, then $N=4.2$ measurements during the 21-day budget period and $E_{Q_I}=\pm 107$ acre-ft (± 0.132 hm^3) per 21 days. Similar computations were made to obtain the outflow error through cross section 9 of $E_{Q_O}=\pm 121$ acre-ft (± 0.149 hm^3) per 21 days. The values for E_{Q_I} and E_{Q_O} are shown in table 2 as sampling errors for water-budget components Q_I and Q_O, respectively.

A comparison of the net change in discharge in reach 1 of -56 acre-ft (-0.069 hm^3) for budget period 688–708 with the average sampling error in this change, computed as $\sqrt{(107)^2+(121)^2}=\pm 162$ acre-ft (± 0.200 hm^3), emphasizes the significance of the sampling error in the discharge components of the water budget.

No independent evaluation of the discharge error was made for the flow at the gaging stations at cross sections 13, 17, and 23, because the flow characteristics and measurement conditions at these stations are similar to those at cross sections 1 and 9 and the frequency of discharge measurements is the same. The error relation of figure 2 was therefore applied to the flow at cross sections 13, 17, and 23.

Of the 12 components included in the water budget, Q_I and Q_O are generally the largest (table 1). A comparison of the ET values in table 1 with their corresponding discharge values suggests that a high ET coincides with a high discharge. But, this is true only during periods of high runoff in the summer when thunderstorms provide sufficient moisture to satisfy the seasonally high potential ET. Thus, all ET values associated with a high runoff in table 1 reflect, in part, a large sampling error and are therefore not considered reliable estimates of the true ET.

The errors in the volume of discharge for those budget periods containing days without streamflow were not evaluated and the ET rates computed for those periods were discarded.

Possible bias in the determinations of Q_I and Q_O was investigated for the gaging stations at cross sections 1 and 9. The difference between the measured discharge and the computed discharge obtained from the stage-discharge rating using the river stage observed on the day of the measurement was plotted against the computed discharge. The plots for both stations show a relatively uniform distribution of points about the line of zero difference, indicating no significant bias in the computed discharges. A further investigation of possible bias in streamflow was made by comparing the gains and losses in the computed flow in reach 1 for days midway between discharge measurements with the gains and losses on days when discharge measurements were made. The gains and losses obtained from the computed discharges appear to be equally distributed among the gains and losses from the measured discharges, thus supporting the assumption of no bias in the computed discharges at cross sections 1 and 9. A plot of these gains and losses with time of year for the period before the phreatophytes were removed indicates that the Gila River was a gaining stream during the winter months (October through February) and a losing stream during the summer months of high ET (March through June). No independent evaluation of streamflow bias was made for reaches 2a or 3.

CHANNEL STORAGE

The change in channel storage within a reach of the Gila River during a budget period is computed from the average change in wetted cross-sectional areas of the channel at the upstream and downstream ends of the reach. This change may be expressed as

$$\Delta C=\left(\frac{A_{I_1}+A_{O_1}}{2}-\frac{A_{I_2}+A_{O_2}}{2}\right)L/43{,}560, \qquad (6)$$

where ΔC is the change in channel storage in acre-feet, L is the length of the river channel within the reach in feet, and A_I and A_O are the wetted cross-sectional areas of the river channel in square feet at the upstream and downstream ends of the reach, respectively. Subscripts 1 and 2 denote the first and last day of the budget period, respectively. A positive change in ΔC indicates a depletion in channel storage and a corresponding addition to ET. A negative change in ΔC indicates an increase in channel storage and a corresponding subtraction from ET.

A_I and A_O in equation 6 were not measured for each budget period but were derived indirectly from a predetermined area-discharge relation. Figure 3 shows the data points defining the relation between measured discharge and corresponding wetted area using data from selected discharge measurements at cross section 9 for water years 1963–69.

Similar relations were defined for cross sections 1, 13, and 17. The curves defining the area-discharge relation for each cross section all lie within the scatter of the data defining the curve for any one cross section. Thus, only one relation approximating the average of all of the curves has been used to obtain the A_I and A_O values in equation 6. This relation is expressed as

$$A=2.9q^{0.65}, \qquad (7)$$

where q is the daily average discharge in cubic feet per second and A is the wetted cross-sectional area in square feet.

The change in channel storage for each budget period was determined by solving equation 7 for A_{I_1}, A_{I_2}, A_{O_1}, and A_{O_2}, using the inflow and outflow discharges computed for the beginning and ending days of the budget period and substituting these area values in equation 6. The discharge values and corresponding cross-sectional areas for budget period 688–708 are given in table 4. Substituting the A_I and A_O values in table 4 into equation 6, where L=36,800 ft (11,200 m) for reach 1, gives ΔC=23 acre-ft (0.028 hm³) per 21 days.

The accuracy of ΔC is dependent on the accuracy of the A_I and A_O values in equation 6. The error in the determination of A may be expressed as

$$E_A=(e_1^2+e_2^2+e_3^2)^{1/2}, \qquad (8)$$

where

- E_A = the total sampling error in the wetted cross-sectional area of the river channel,
- e_1 = the temporal variability of A at a given cross section as indicated by the scatter of data points in the q versus A relation of figure 3,
- e_2 = the variability of A between cross sections as indicated by the difference in the q versus A curves of figure 3,
- e_3 = the variability in A due to the error in the discharge used in equation 7 to obtain A as indicated by the q versus e_q relation of figure 2.

Equation 8 is applicable only if e_1, e_2, and e_3 are independent estimates of error. This assumption is considered valid because no relation should be expected to exist between these three error terms.

The temporal variability in A at cross section 9 ranges from ±10 ft² (±0.93 m²) when q=10 ft³/s (0.28 m³/s) to ±70 ft² (±6.50 m²) when q=4,000 ft³/s (113 m³/s) (fig. 3)

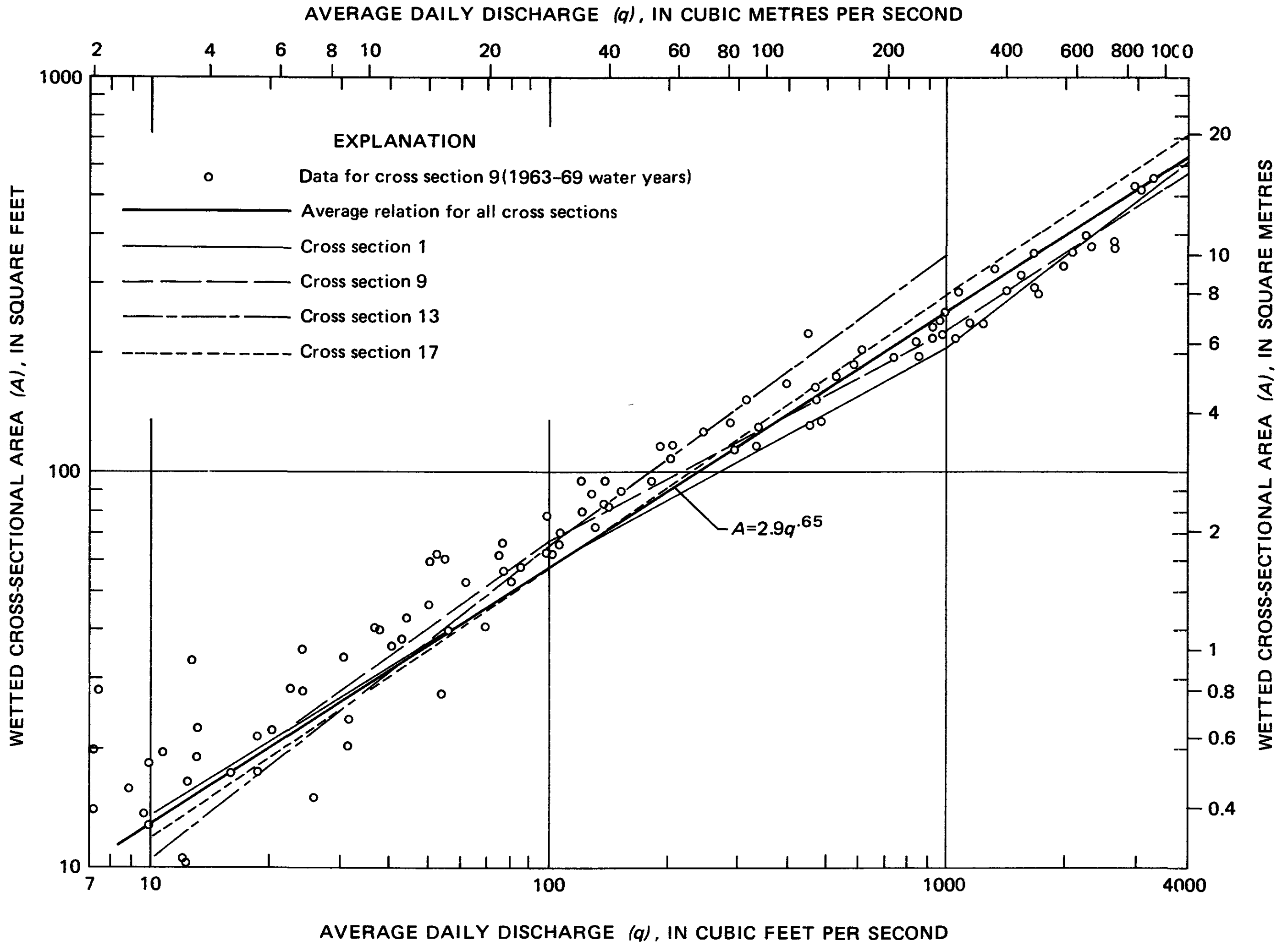

FIGURE 3.—Relation between average daily discharge for the budget period and wetted cross-sectional area for cross sections 1, 9, 13, and 17.

TABLE 4.—*Average daily discharge* (q), *corresponding wetted cross-sectional area of river channel* (A), *and error in the area* (E_A) *for cross sections 1 and 9 on budget period days 688 and 708*

Budget day	Cross section 1			Cross section 9		
	q (ft³/s)	A_I (ft²)	E_A (ft²)	q (ft³/s)	A_O (ft²)	E_A (ft²)
688	47.0	35.4	17.5	52.0	37.8	18.2
708	7.0	10.3	8.3	5.1	8.4	7.4

and can be expressed as $e_1=4.6q^{0.33}$. The variability in A between cross sections ranges from $\pm$ 2 ft² ($\pm$0.2 m²) at q=10 ft³/s (0.28 m³/s) to $\pm$55 ft² ($\pm$5.1 m²) at q=4,000 ft³/s (113 m³/s) (fig. 3) and can be expressed as $e_2=0.38q^{0.60}$. The error in A attributed to the error in the computed discharge can be derived from the q versus e_q relation of figure 2. For this analysis the fractional error, e_q, is assumed to be 0.13 for all discharges, and so the error in q is $0.13q$. When this discharge is included in equation 7, e_3 can be expressed in terms of the expected values:

$$e_3^2=\epsilon\left[2.9(q+0.13q)^{0.65}\right]^2-\left[\epsilon(2.9q^{0.65})\right]^2. \quad (9)$$

Applying a first order approximation of the Taylor series expansion to equation 9 and dropping all but the first terms in the series yields $e_3=0.17q^{0.65}$. Inclusion of higher order terms in the Taylor series was found to have no effect on the expression for e_3.

The above expressions for e_1, e_2, and e_3, when substituted in equation 8, can be approximated by

$$E_A=3.9q^{0.39}, \quad (10)$$

where E_A is in square feet and q is in cubic feet per second.

Figure 4 shows the relation of q to E_A both as defined

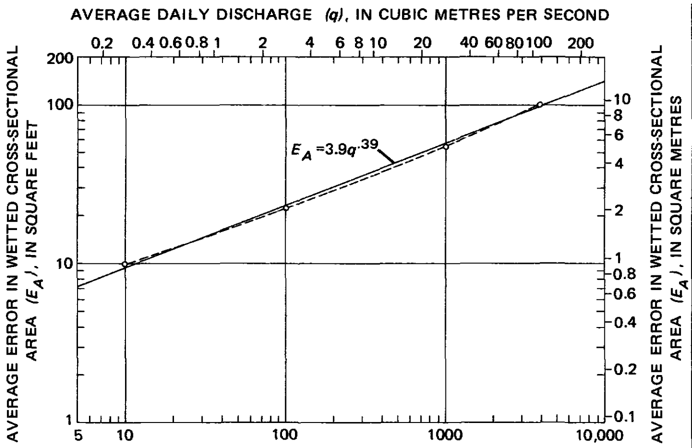

FIGURE 4.—Relation between the average daily discharge for a budget period and the error in wetted cross-sectional area of the Gila River channel.

by equation 8 and as defined by equation 10. Equation 10 was used to obtain the errors in the wetted cross-sectional areas shown in table 4.

An estimate of the standard error in ΔC for any given budget period can be obtained from the square root of the sum of the variances of each A_I and A_O value in equation 6 or

$$E_{\Delta C} = \frac{L}{2 \times 43{,}560}\left(E^2_{A_{I_1}} + E^2_{A_{O_1}} + E^2_{A_{I_2}} + E^2_{A_{O2}}\right)^{1/2}, \quad (11)$$

where the E_A terms are in square feet and $E_{\Delta C}$ is in acre-feet. Solving equation 11 for the E_A values in table 4 gives $E_{\Delta C} = \pm$ 12 acre-ft ($\pm$ 0.015 hm^3) for budget period 688–708. This error is about 50 percent of the computed change in channel storage of +23 acre-ft (+0.028 hm^3) but is only 2 percent of the total *ET* of 513 acre-ft (0.633 hm^3).

ΔC is generally not a significant component in the water-budget equation, and greater refinement in its computation was not considered justified. The determination of $E_{\Delta C}$ in equation 11 assumes independence of the error terms E_{A_I} and E_{A_O} both with time and between cross sections.

The computed increase in channel storage ($-\Delta C$ in equation 1) is substantially underestimated during periods of high discharge when low-lying portions of the channel banks are overtopped and surface water goes into depression storage in the many small channels and low areas on the flood plain. *ET* values computed for these periods are commonly unrealistically high and actually indicate a large component of unmeasured water going into storage. This may partly explain the unrealistically high *ET* values for the budget periods ending on project days 394, 666, 687, and 729 (table 1)—all periods of high discharge in the Gila River. The subsequent period of drainage from depression storage immediately following a high discharge frequently causes an underestimate of the decrease in channel storage ($+\Delta C$ in equation 1) resulting in computed *ET* values which are too low. Reliable field measurements of depression storage were not possible, particularly during high flow periods. However, the errors in discharge of the Gila River for these high flow periods are too large to give reliable estimates of *ET* and those *ET* values are generally disregarded.

TRIBUTARY INFLOW

Runoff from tributaries adjacent to the study area originates from 225 mi^2 (583 km^2) of drainage area. A total of 43 tributaries draining 95 percent of the area adjacent to reaches 1 and 2 were instrumented with either recording-stage gages or crest-stage gages. The stage data from the recording gages were used for estimating runoff volumes, and the crest-stage gages were used primarily to define periods of significant runoff. Tributary runoff into reach 3 was not measured, because collection of water-budget data on this reach was discontinued before instrumentation on the tributaries was fully established.

Normally the tributaries were not monitored during the winter season (November through April) because precipitation during this period is generally from frontal storm systems which may cover a large area but seldom produce significant flow volumes into the Gila River. Tributary inflow to the study area was observed during a few large winter storms, but these periods coincide with a high discharge in the Gila River and a corresponding large error in the water budget. The *ET* values for these periods have been discarded or are recognized as not reliable. The only significant tributary runoff observed in the project area during the 9-year study occurred from May through October. Most of this runoff resulted from short, intense thunderstorms in July and August.

Tributary runoff occurred, on the average, less than 4 percent of the time, or about 13 days out of the year, and runoff in any one tributary occurred, on the average, only 3 days per year. Tributary runoff occurred in about 30 of the over 180 budget periods evaluated during the nine-year study, but only 15 of these periods had runoff volumes which were a significant part of the *ET*.

Estimates of the volume of tributary runoff into reaches 1, 2, and 2a were obtained from stage-discharge relations and peak discharge-storm volume relations developed for each tributary by Burkham (1976). The runoff volumes during budget period 688–708 in each of the 20 tributaries bordering reach 1 (table 5) were estimated from these relations.

TABLE 5.—*Tributary runoff into reach 1 during budget period 688–708*

Tributary number	Runoff (acre-ft)	Tributary number	Runoff (acre-ft)
24	----	34	7.0
25	----	35	14.0
26	----	36	2.0
27	----	37	5.0
28	0.4	38	50.0
29	----	38.5	----
30	----	39	42.0
31	3.0	40	2.5
32	5.0	41	1.4
33	----	42	120.0
Subtotal	8.4		243.9
Total			252.3

Burkham (1976) indicated that definition of the stage-discharge and peak discharge–storm volume relations are poor at best and estimated that the computed volume from a runoff event in any one tributary may be in error by 100 percent; however, periods when runoff did not occur were considered to be accurately defined. Because of the generally low volume of tributary inflow to the study reaches and their relatively infrequent occurrence, no evaluation was made of their standard error. All estimates of tributary inflow to the study area were thus assumed to be 100 percent in error. Accordingly, the tributary runoff of Q_T=252 acre-ft (0.311 hm^3) for budget period 688–708 (table 2) was assumed to have an error of $E_{Q_T}=\pm 252$ acre-ft (±0.311 hm^3).

PRECIPITATION

Accumulated precipitation during each budget period was obtained from wedge gages located at the ends of each cross section (see fig. 1). Visits to the gages were made at two- or three-week intervals which coincided (within two or three days) with the last day of the budget period and with the field measurements of soil moisture. In a few instances precipitation occurred during the two-day period required to visit all the gages in a reach, resulting in discrepancies in the total accumulated precipitation between gages for the budget period. These occurrences were rare, and an attempt was made to correct only the obvious discrepancies.

Each gage was assigned a portion of the total area in the reach using a method of proportioning which closely approximates the Thiessen method (1911). The total accumulated precipitation for the budget period was computed as an average weighted value from

$$\overline{P}=\left(\sum_{j=1}^{n} A_jP_j\right)\Big/\sum_{j=1}^{n} A_j , \qquad (12)$$

where

$\overline{P}$ = the average weighted precipitation for the budget period, in inches,

P_j = the accumulated precipitation at gage j for the budget period, in inches,

A_j = the area assigned to gage j, in acres, and

n = total number of gages in the reach.

Table 6 shows the precipitation amounts observed at each gage in reach 1 for budget period 688–708 and the areas assigned to each gage. The average weighted precipitation of these 10 gages is $\overline{P}$=1.21 in. (30.7 mm), or 174 acre-ft (0.215 hm^3) in volume for the budget period.

Occasionally the precipitation at a gage was not obtained. In such instances, the precipitation was estimated using observed data from nearby wedge gages or from the recording gages located at the ends of the reach. Thus, all budget periods contain a complete set of data.

The total measurement error in the computed average precipitation for a budget period can include both a bias error and a sampling error. A bias error commonly occurs when the gage is located too close to trees, buildings, or other obstructions which interfere with catchment in the gage. This type of error is not considered significant for the project area as all of the wedge gages were located in areas of ample exposure. The measurement of precipitation by the gages may have been slightly low during the summer months owing to loss by evaporation from the gage; however, a thin film of oil was maintained in each gage to minimize evaporation, and this loss is not considered significant.

TABLE 6.—*Area assigned to each precipitation gage in reach 1 and precipitation amounts observed at each gage during budget period 688–708*

	Gage number									
	0101	0106	0307	0312	0513	0518	0719	0724	0925	0930
Area (acres)	154	103	77	371	55	342	305	129	92	95
Precipitation (in.)	1.50	1.58	1.60	1.25	1.40	1.00	1.10	1.10	1.15	1.15

Total area reach 1=1,723 acres
Average weighted precipitation reach 1, $\overline{P}$=1.21 inches or 174 acre-ft.

Thus, all bias errors involved in the measurement of precipitation are believed to be negligible.

The sampling error in the measurement of precipitation may be attributed to missing data and insufficient sampling points (gages) within the reach. Included in the sampling error is the human error resulting from misreading the gage and the instrument error resulting from leakage of a damaged gage or debris falling into the gage.

In this analysis the missing-data error was first evaluated. The rate of change in the missing-data error as the number of sample points increases was then evaluated to obtain an estimate of the sampling error for a complete set of sample points—that is, with no missing data.

The missing-data error of the average precipitation per budget period decreases as the number of gages monitoring precipitation on the reach increases. Estimates of this error were obtained for reach 1 by evaluating the departure of the average precipitation computed using all gages in the reach ($n=10$ gages) from the average precipitation computed using m gages, where $m<n$. This analysis was made using a total of 114 budget periods with average precipitation amounts of 0.06 in. (1.5 mm) or more. The data were subdivided into three ranges—0.06 to 0.50 in. (1.5–12.7 mm), 0.51 to 1.50 in. (13.0–38.1 mm), and greater than 1.50 in. (38.1 mm)—and sampling errors were evaluated for each of the three ranges. Of the 114 periods, 53 fell within the lower range defining an average precipitation of 0.31 in. (7.9 mm), 45 periods fell within the midrange defining an average precipitation of 0.85 in. (21.6 mm), and 16 periods fell within the higher range defining an average precipitation of 2.06 in. (52.3 mm).

The following six steps outline the procedure used in evaluating the missing-data error, $\bar{S}_m$. The procedure assumes that the observations of precipitation at each gage are independent and that $\bar{S}_m=0$ when $m=10$ gages. The computations described below refer to the precipitation data in a given range; these same computations were made for each range of data:

1. Compute the average unweighted precipitation, $\bar{P}_t$, for each budget period, t, from

$$\bar{P}_t=\frac{1}{10}\sum_{j=1}^{10} P_{jt}\ , \tag{13}$$

where P_{jt} is the accumulated precipitation at gage j for budget period t. Table 7 gives a generalized example of the array of P_{jt} data and resulting $\bar{P}_t$ values involved in this step.

2. Compute the average departure of precipitation at each gage from the average precipitation computed from all gages in the reach from

$$\bar{R}_j=\frac{1}{k}\sum_{t=1}^{k} R_{jt}\ , \tag{14}$$

where

$\bar{R}_j$ = the average departure of precipitation at gage j, for k budget periods, and

$$R_{jt}=\bar{P}_t-P_{jt}. \tag{15}$$

The R_{jt} and $\bar{R}_j$ values defined by this step are included in table 7. The $\bar{R}_j$ values computed for each of the 10 gages in reach 1 are given in table 8.

3. Compute the standard deviation of each $\bar{R}_j$ value from

$$s_j=\left[\frac{\sum_{t=1}^{k}(R_{jt})^2-\left(\sum_{t=1}^{k}R_{jt}\right)^2\Big/k}{k-1}\right]^{\frac{1}{2}}\ , \tag{16}$$

where s_j is the standard deviation of the average departure in precipitation for gage j based on precipitation data from k budget periods. Table 7 includes an example of the array of s_j values defined by this step, and table 8 lists the s_j values for each of the 10 gages in reach 1.

4. For each of $m=1, \ldots, 9$ gages, arrange the gages into 10 unique permutations ($p=10$) and for each

TABLE 7.—*Example of precipitation data array used to compute the average precipitation for each budget period* ($\bar{P}_t$), *the departure of precipitation at a given gage from the average precipitation* (R_{jt}), *the average departure for all budget periods* ($\bar{R}_j$), *and the standard deviation of the average departure* (s_j)

Gage (j)	Budget period (t) 1	2	. . .	k	$\bar{R}_j$	s_j
1	$P_{1,1}$	$P_{1,2}$	. . .	$P_{1,k}$		
	$R_{1,1}$	$R_{1,2}$		$R_{1,k}$	$\bar{R}_1$	s_1
2	$P_{2,1}$	$P_{2,2}$	. . .	$P_{2,k}$		
	$R_{2,1}$	$R_{2,2}$		$R_{2,k}$	$\bar{R}_2$	s_2
.	.	.	. . .	.	.	.
n	$P_{n,1}$	$P_{n,2}$	. . .	$P_{n,k}$		
	$R_{n,1}$	$R_{n,2}$		$R_{n,k}$	$\bar{R}_n$	s_n
$\bar{P}_t$	$\bar{P}_1$	$\bar{P}_2$	. . .	$\bar{P}_k$		

TABLE 8.—*Average departure in precipitation of each gage in reach 1* ($\bar{R}_j$) *and the standard deviation of the average departure* (s_j) *computed for three precipitation ranges with each range containing* k *budget periods of data*

[All values are in inches per 21 days]

Precipitation range		0.06 to 0.50 inches		0.51 to 1.50 inches		1.51 inches or greater	
Sample size		k=53 periods		k=45 periods		k=16 periods	
j	Gage No.	$\bar{R}_j$	s_j	$\bar{R}_j$	s_j	$\bar{R}_j$	s_j
1	0101	−0.001	±0.156	−0.007	±0.321	0.046	±0.511
2	0106	−.003	.180	−.079	.261	−.109	.384
3	0307	−.002	.140	.012	.288	.045	.439
4	0312	.005	.182	−.095	.252	.015	.344
5	0513	.002	.160	.030	.289	.033	.381
6	0518	.006	.163	−.018	.272	−.032	.567
7	0719	.002	.176	.002	.302	.001	.618
8	0724	−.030	.127	.020	.337	.032	.636
9	0925	.019	.161	.049	.342	−.001	.556
10	0930	.001	.155	.086	.350	−.029	.438

permutation (v=1, . . ., 10) compute the mean departure in the average precipitation at m gages for all m=1, . . ., 9 by the equation

$$\bar{\bar{R}}_{mv}=\frac{1}{m}\sum_{j=1}^{m}\bar{R}_{jv} \quad , \qquad (17)$$

where $\bar{\bar{R}}_{mv}$ is the mean departure for the m gages in permutation v. A generalized example of the array of $\bar{\bar{R}}_{mv}$ values defined by this step is shown in table 9.

5. For each of the v=1, . . ., 10 permutations, compute an approximation of the standard deviation of the mean departure of m gages for all m=1, . . ., 9 by the equation

$$S_{m,v}=\frac{1}{m}\left(s^2_{1,v}+s^2_{2,v}+\cdots+s^2_{m,v}+2rs_{1,v}s_{2,v} + 2rs_{1,v}s_{3,v}+\cdots+2rs_{1,v}s_{m,v} + \cdots +2rs_{(m-1),v}s_{m,v}\right)^{1/2} \quad , \qquad (18)$$

where $S_{m,v}$ is the standard deviation of the mean departure for m gages arranged in permutation v, $s_{1,v} \ldots s_{m,v}$ are the standard deviations in the departures for $1 \ldots m$ gages, respectively, in permutation v, and r is the correlation coefficient of precipitation measured at any two gages. Equation 18 assumes that the population mean and variance of the departures in precipitation are constant throughout the reach. Furthermore, the $\bar{R}_j$ values were computed in such a manner that

$$\sum_{j=1}^{n}\bar{R}_j=0, \qquad (18a)$$

TABLE 9.—*Example of array of mean departures of precipitation* ($\bar{\bar{R}}_{mv}$) *and standard deviations of the mean departures* (S_{nv})

Number of gages	Permutations (v)						
	v=1		v=2		. . .	v=p	
	$\bar{\bar{R}}_{m,1}$	$S_{m,1}$	$\bar{\bar{R}}_{m,2}$	$S_{m,2}$	. . .	$\bar{\bar{R}}_{m,p}$	$S_{m,p}$
m=1	$\bar{\bar{R}}_{1,1}$	$S_{1,1}$	$\bar{\bar{R}}_{1,2}$	$S_{1,2}$	. . .	$\bar{\bar{R}}_{1,p}$	$S_{1,p}$
m=2	$\bar{\bar{R}}_{2,1}$	$S_{2,1}$	$\bar{\bar{R}}_{2,2}$	$S_{2,2}$	. . .	$\bar{\bar{R}}_{2,p}$	$S_{2,p}$
.	.	.	.	.	. . .	.	.
m=$n-1$	$\bar{\bar{R}}_{n-1,1}$	$S_{n-1,1}$	$\bar{\bar{R}}_{n-1,2}$	$S_{n-1,2}$	. . .	$\bar{\bar{R}}_{n-1,p}$	$S_{n-1,p}$

and thus defining a "closed array" of departures. Chayes (1971, p. 40) showed that a closed array of values having a constant mean and variance defines a correlation coefficient which is negatively biased. This coefficient, r, can be approximated as a constant value from the relation

$$r=\frac{1}{1-n} \quad , \qquad (19)$$

where n is the total number of gages in the reach. For reach 1, r=−0.11 with n=10 gages. Justification for using equation 19 to approximate the true correlation coefficient was investigated by evaluating $S_{m,v}$ in equation 18 for selected combinations of gages using an estimate of the true correlation coefficient, $\rho_{j_1 j_2}$, computed from the relation

$$\rho_{j_1 j_2}=\sum_{t=1}^{k}\left(R_{j_1 t}R_{j_2 t}\right)\Big/\left(\sum_{t=1}^{k}R^2_{j_1 t}\ \sum_{t=1}^{k}R^2_{j_2 t}\right)^{1/2} \quad , \qquad (20)$$

where $\rho_{j_1 j_2}$ is the correlation coefficient between gages j_1 and j_2 and $R_{j_1 t}$ and $R_{j_2 t}$ are as defined by equation 15. A comparison of the $S_{m,v}$ values calculated using $\rho_{j_1 j_2}$ with the $S_{m,v}$ values calculated using $r=-0.11$ showed close agreement, thus supporting the use of equation 19 to approximate the correlation coefficient.

The $\bar{\bar{R}}_{mv}$ and S_{mv} values for the upper precipitation range are shown in table 10 for two of the $p=10$ permutations used in this analysis. The gages are arranged in sequences of increasing s_j (see table 8) for permutation v_1 and decreasing s_j for permutation v_2.

6. Compute the average standard deviation ($\bar{S}_m$) of the 10 permutations of $\bar{\bar{R}}_{mv}$ for each m using the relation

$$\bar{S}_m = \left[\frac{1}{p} \left(\sum_{v=1}^{p} \bar{\bar{R}}^2_{mv} + \sum_{v=1}^{p} S^2_{mv} \right) \right]^{1/2}, \qquad (21)$$

where $\bar{\bar{R}}_{mv}$ and S_{mv} are defined in equations 17 and 18, respectively, subscript v is a given permutation of gages, and $p=10$ is the total number of permutations. The departure, $\bar{\bar{R}}_{mv}$, is included in equation 21 to account for the average departure in the precipitation error; however, $\bar{\bar{R}}_{mv}$ is of little significance as indicated in table 10 and could have been omitted.

The $\bar{S}_m$ values defined in equation 21 represent the average missing-data error associated with m gages. A plot of these values is shown in figure 5 for each of the three precipitation ranges with a solid curve drawn through the points to approximate the average rate of decrease in $\bar{S}_m$ as m increases. The broken curves bounding each solid curve approximate the maximum and minimum missing-data errors computed from the 10 permutations.

Table 11 lists the $\bar{S}_m$ values obtained from the curves in figure 5 for each of the three precipitation ranges investigated. These $\bar{S}_m$ values show the expected departure of an average precipitation value computed from data at m gages from the average value computed

TABLE 10.—*$\bar{\bar{R}}_{mv}$ and S_{mv} values for precipitation in the upper range $\bar{P} \geq$ 1.51 in. (38.4 mm) in reach 1 with gages arranged in order of increasing s_j and in order of decreasing s_j*

m	Gages in sequence of increasing s_j: Gage No.	$\bar{\bar{R}}_{mv_1}$ (in.)	S_{mv_1} (in.)	Gages in sequence of decreasing s_j: Gage No.	$\bar{\bar{R}}_{mv_2}$ (in.)	S_{mv_2} (in.)
1	0312	0.015	±0.344	0724	0.032	±0.636
2	0513	.024	.166	0719	.016	.312
3	0106	−.020	.187	0518	0	.290
4	0930	−.022	.130	0925	0	.224
5	0307	−.009	.114	0101	.009	.114
6	0101	0	.149	0307	.015	.087
7	0925	0	.124	0930	.009	.080
8	0518	−.004	.078	0106	−.006	.041
9	0719	−.004	.071	0513	−.002	.038

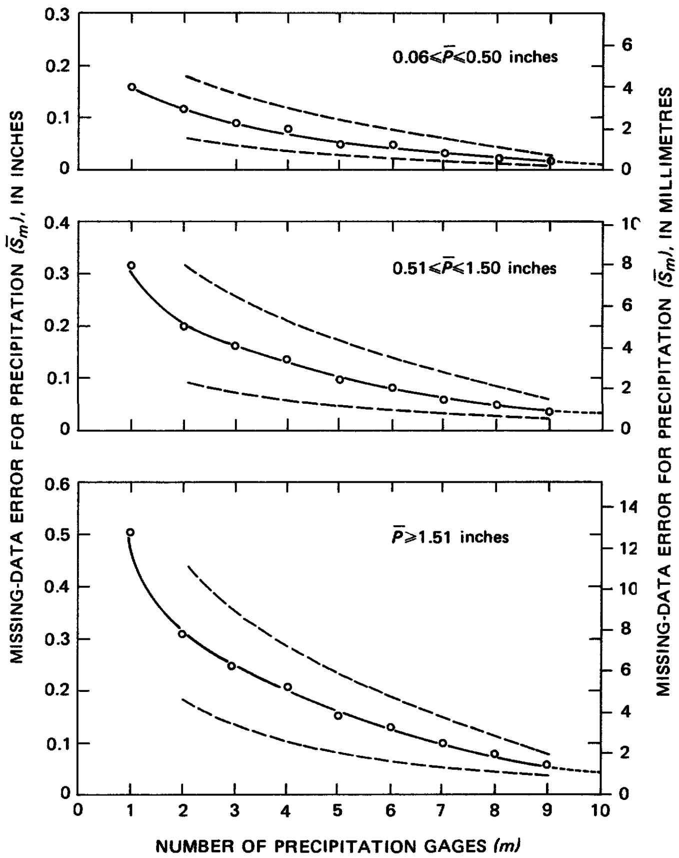

FIGURE 5.—Average relation between the number of precipitation gages and the missing-data error of average precipitation during the budget period for three ranges of precipitation in reach 1.

from data at 10 gages. For example, referring to table 11, the departure of the average precipitation defined with 5 gages from the average precipitation defined with 10 gages can be expected to have a mean standard deviation of $\bar{S}_5=\pm0.10$ in. (±0.25 cm) when the average precipitation is in the middle range.

The development of the curves in figure 5 assumes that no error exists in the average precipitation computed from 10 gages. Each curve shows, however, a residual error ($\bar{S}_{10}$) when $m=10$ gages. This residual error is attributed to two factors: (1) the use of a constant r to approximate the actual correlation coefficients and (2) differences in the standard deviations, s_j, of the gages (table 8). These curves therefore overestimate the expected standard deviation of an average precipitation value computed from $m \leq 10$ gages.

The primary objective of the analysis of precipitation error is not to determine the error attributed to missing data, as all budget periods contain a complete set of data, but rather to define the sampling error in precipitation computed from a complete set of data ($m=10$ gages). This sampling error was estimated by evaluating the rate of change in $\bar{S}_m$ as m approaches 10

TABLE 11.—*Average missing-data errors* ($\bar{S}_m$) *of precipitation in each precipitation range from curves in figure 5 for* m=*1 to 10 gages in reach 1. Included are the unadjusted* (∇) *and adjusted* ($E_{\bar{P}}$) *sampling errors for* m= *10 gages*

Precipitation range (in.)	Average precipitation (in.)	Number of periods (*k*)	Average standard deviation of $\bar{R}_{mv}$ (from curves in fig. 5, in inches)										Sampling error (in.)	
			$\bar{S}_1$	$\bar{S}_2$	$\bar{S}_3$	$\bar{S}_4$	$\bar{S}_5$	$\bar{S}_6$	$\bar{S}_7$	$\bar{S}_8$	$\bar{S}_9$	$\bar{S}_{10}$	Unadjusted ∇	Adjusted $E_{\bar{P}}$
0.06–0.50	0.31	53	0.16	0.12	0.09	0.08	0.05	0.05	0.04	0.03	0.02	0.02	±0.054	±0.056
0.51–1.50	.85	45	.31	.20	.17	.14	.10	.09	.07	.05	.04	.03	.092	.097
>1.50	2.06	16	.50	.31	.25	.21	.16	.13	.10	.08	.06	.04	.134	.141

(fig. 5). This sampling error, ∇, when expressed in terms of the gradient between $\bar{S}_m$ and $\bar{S}_{2m}$ is of the general form

$$a\nabla^4+b\nabla^2+c=0, \quad (22)$$

where

$$a=\sum_{m=1}^{n'/2}\left(\bar{S}^2_m-\bar{S}^2_{2m}\right), \quad (23)$$

$$b=\sum_{m=1}^{n'/2}\left(4\bar{S}^2_m\bar{S}^2_{2m}-3\bar{S}^2_{2m}-\bar{S}^4_m\right), \quad (24)$$

$$c=\sum_{m=1}^{n'/2}\left(3\bar{S}^2_m\bar{S}^2_{2m}-2\bar{S}^6_{2m}-\bar{S}^4_m\bar{S}^2_{2m}\right), \quad (25)$$

and

where n' is the largest even number ⩽n. The derivations of equation 22 and the coefficients *a*, *b*, and *c* are given in the section "Development of equations describing unadjusted sampling error, ∇."

The coefficients *a*, *b*, and *c* were determined for each of the three ranges of precipitation by substituting the appropriate $\bar{S}_m$ and $\bar{S}_{2m}$ values of table 11 in equations 23, 24, and 25. These coefficients were then used to solve equation 22 for ∇, which defines the sampling error of precipitation for $m=10$ gages. This value does not include the residual error, $\bar{S}_{10}$, (table 11) and is therefore referred to as the "unadjusted" sampling error as shown in table 11.

The total "adjusted" sampling error, $E_{\bar{P}_m}$, in precipitation for any given number of gages includes both the missing-data error ($\bar{S}_m$) and the sampling error, ∇. This total error is computed from

$$E_{\bar{P}_m}=\left(\bar{S}^2_m+\nabla^2\right)^{1/2}, \quad (26)$$

where $\bar{S}_m$ is defined by equation 21 and ∇ is defined by equation 22. The total adjusted sampling error for a complete set of data ($m=10$ gages) was obtained for each precipitation range by substituting the appropriate ∇ and $\bar{S}_{10}$ values of table 11 into equation 26. These $E_{\bar{P}}$ values are included in table 11 and are plotted against their corresponding average precipitation values in figure 6.

A curve closely approximating the relation between these sampling errors and their respective average precipitation values may be expressed as

$$E_{\bar{P}}=0.10\bar{P}^{0.47}, \quad (27)$$

where $E_{\bar{P}}$ is the sampling error in precipitation in inches for 10 gages and $\bar{P}$ is the average precipitation in inches.

Substituting the average precipitation for the example period 688–708 of $\bar{P}=1.21$ in. (30.7 mm) (see table 6) into equation 27 gives a total adjusted sampling error of $E_{\bar{P}}=\pm0.11$ in. (±2.8 mm) or ±16 acre-ft (±0.020 hm³).

Independent evaluations of the precipitation errors for reaches 2, 2a, and 3 were not made, but the error relation of figure 6 is considered applicable to these reaches because the gage density of these reaches is similar to that for reach 1.

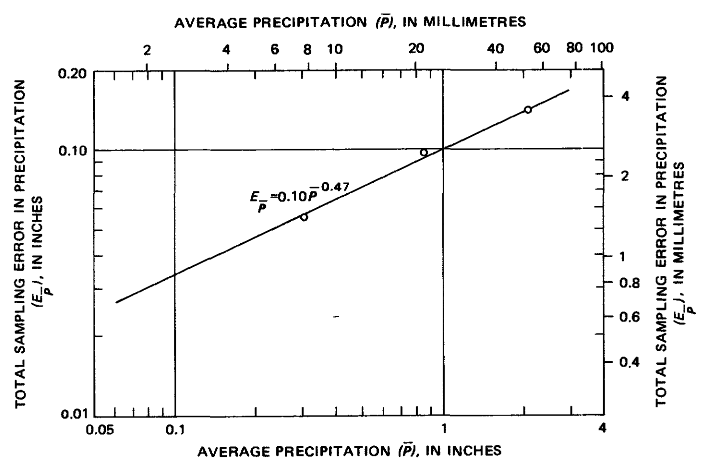

FIGURE 6.—Relation between average precipitation during the budget period and the total adjusted sampling error in the average precipitation.

A few conclusions can be drawn from this analysis regarding optimum gage density and variation in the areal distribution of precipitation on the study area. Referring to figure 5, the missing-data error curves show a relatively small rate of change in error as m approaches 10 gages. This suggests that a large increase in gage density above 10 gages on reach 1 may not significantly reduce the error in computing average precipitation. In fact, precipitation per budget period in the two lower ranges can be defined from only six or seven gages with little loss in accuracy. Also, as noted previously, the average precipitation for a budget period is a weighted value reflecting the fraction of total area in the reach assigned to each gage. For purposes of simplicity in the error analysis, precipitation at each gage was given equal weight using equation 13. A comparison of average precipitation values weighted by area (equation 12) with the equally weighted values indicates no significant differences. Equation 27, which defines the error of an unweighted precipitation value, is therefore considered applicable to the weighted precipitation values in the water budget.

Finally, a correlation between precipitation on the left bank of the flood plain with precipitation on the right bank of the flood plain indicates that precipitation averages slightly greater (+0.02 in. or +0.5 mm per budget period) on the left bank. This is attributed to the orographic position of Mount Turnbull and the Santa Teresa Mountains which rise to an altitude of over 8,000 ft (2,400 m) 8 miles (13 km) to the south (left bank) of the flood plain.

SOIL-MOISTURE CONTENT

The soil-moisture content was measured within two areas of each reach: (1) the flood plain which corresponds with the area for which ET is evaluated and (2) the adjacent terrace area which extends out from the flood plain to the contact of the saturated terrace alluvium with the basin fill. Measurements were made with a neutron probe at two- to three-week intervals thus defining the water-budget periods. The difference in the moisture content measured between the beginning and end of the period defines the change in moisture content for the budget period.

An access hole for measuring moisture content was located within about 15 ft (4.6 m) of each ground-water observation well in the study area. Each hole was classified as one of the following three types: (1) river hole located adjacent to the river, (2) flood-plain hole located between the river and the terrace, or (3) terrace hole located in the adjacent terrace. The river and flood-plain holes were used to obtain the change in moisture content in the unsaturated zone of the flood-plain alluvium and the terrace holes were used to obtain the change in the unsaturated zone of the adjacent terrace alluvium. A detailed description of the installation of the access holes was given by Myrick in Culler and others (1970).

Neutron probes were used to obtain moisture-content readings in each hole at the ½-ft (0.15 m) and 1-ft (0.30 m) depths below the land surface and at 1-ft (0.30 m) intervals throughout the remaining depth of the hole which generally extended several feet below the ground-water level. The change in moisture content was determined for three zones of the profile: (1) the soil zone extending from the land surface to 2½ ft (0.76 m) below land surface in the flood plain and to 5 ft (1.52 m) below land surface in the terrace, (2) the intermediate zone extending from the bottom of the soil zone to about 3 ft (0.9 m) above the highest observed ground-water level, and (3) the capillary zone extending from the bottom of the intermediate zone to the bottom of the hole in the flood plain and to about 3 ft (0.9 m) below the lowest observed ground-water level in the terrace. No intermediate zone was defined for the flood plain of reach 1 because of the relatively shallow ground-water level in the reach.

The change in moisture content in each of these three zones within the ET area of the flood plain corresponds to the water-budget components $\Delta\overline{M}_S$, $\Delta\overline{M}_I$, and $\Delta\overline{M}_C$ respectively in equation 1 and items 6, 7, and 8 respectively in table 2. The change in moisture content in the capillary zone of the terrace corresponds to $\Delta\overline{M}_{TC}$ in equation 1 and item 12 in table 2. Moisture changes in the soil and intermediate zones of the terrace ($\Delta\overline{M}_{TS}$ and $\Delta\overline{M}_{TI}$ respectively) are not included in the water-budget equation when evaluating ET from the flood plain because the moisture in these two zones is considered to be removed solely by the overlying terrace vegetation which lies outside the boundaries of the ET area. Moisture in the capillary zone of the terrace, however, is believed to be too deep (20 to 40 ft or 6 to 12 m below land surface) to be readily extracted by the overlying terrace vegetation. Moisture changes in the terrace capillary zone are thus assumed to result from changes in ground-water levels in the adjacent flood-plain alluvium. All significant movement of water out of the terrace capillary zone is assumed to be lateral and in the direction of the flood plain in response to an overall drop in ground-water levels with a general water-level gradient towards the Gila River. All significant movement of water into the terrace capillary zone is also assumed to be lateral but originating from the flood plain in response to an overall increase in ground-water levels with a general water-level gradient away from the river.

The average change in moisture content in a given zone of the reach for a budget period was computed from

$$\Delta\overline{M}_{zt} = \sum_{j=1}^{n} \left(\Delta M_{zjt}\, A_j\right) \Big/ \sum_{j=1}^{n} A_j \ , \tag{28}$$

where

$\Delta\overline{M}_{zt}$ = average weighted change in moisture content in zone z of the reach during budget period t,

$\Delta M_{zjt} = M_{zj\,(t-1)} - M_{zjt}$, (29)

$M_{zj\,(t-1)}$ and M_{zjt} = measured moisture content in zone z of hole j at the beginning $(t-1)$ and end (t) of the budget period,

A_j = surface area assigned to hole j, and

n = total number of access holes in the reach.

The surface area, A_j, assigned to each hole was determined using the same approximation of the Thiessen method that was applied in assigning areas to the precipitation gages. When moisture-content data were missing for an access hole, the change in moisture content for the hole was approximated using the average unweighted change computed from the measured access holes in the reach of the same type (river, flood plain, or terrace) as the unmeasured hole. Table 12 gives the moisture-content data for the soil and capillary zones of each hole in the flood plain of reach 1 measured on budget period days 688 and 708. A negative change in moisture content indicates an increase of moisture in the profile (negative ET component) during the budget period, whereas a positive change indicates a loss of moisture in the profile (positive ET component). The $\Delta\overline{M}_S$ and $\Delta\overline{M}_C$ values shown in inches of moisture-content change at the bottom of table 12 were converted to acre-feet in table 2. Similar computations were made to obtain the $\Delta\overline{M}_{TC}=-44$ acre-ft (-0.054 hm^3) for the terrace capillary zone as shown in table 2.

The soil-moisture data in table 12 indicate that the amount of moisture change during a budget period is relatively small compared to the total moisture measured in the profile. For example, the total moisture content measured in the flood plain of reach 1 averages about 3 in. (8 cm) in the soil zone and about 28 in. (71 cm) in the capillary zone giving a total of 31 in. (79 cm) in the soil profile. The measured change in this moisture for budget period 688–708 includes 0.059 in. (0.150 cm) in the soil zone and 0.575 in. (1.460 cm) in the capillary zone giving a total change of 0.634 in. (1.610 cm), or only 2 percent of the total moisture measured in the reach. Thus, a reliable estimate of this comparatively small moisture change requires that measurements of the total average moisture be highly accurate—allowing for only a fraction of a percent error.

During periods of low streamflow in the Gila River, the moisture-storage components $\Delta\overline{M}_S$, $\Delta\overline{M}_C$, and $\Delta\overline{M}_{TC}$ are generally the most significant components of the

TABLE 12.—*Soil-moisture content measured in the soil and capillary zones of flood plain access holes in reach 1 for budget period days 688 and 708*

[All moisture content values in inches]

Hole type[1]	Hole No.	Area (acres)	Soil zone			Capillary zone		
			$M_{S,\,688}$	$M_{S,\,708}$	ΔM_S	$M_{C,\,688}$	$M_{C,\,708}$	ΔM_C
2	0102	99.2	2.32	2.86	−0.54	20.21	20.58	−0.37
1	0103	55.1	1.79	1.73	.06	43.30	40.28	3.02
1	0104	33.1	3.46	3.30	.16	28.99	27.84	1.15
2	0105	40.4	4.20	4.58	−.38	64.23	64.81	−.58
2	0106	29.4	----	4.20	[2]−.05	----	29.57	[3].13
2	0308	40.4	3.19	2.67	.52	39.43	42.09	−2.66
1	0309	36.7	2.72	2.05	.67	14.66	16.04	−1.38
1	0310	77.1	2.33	1.75	.58	38.91	38.65	.26
2	0311	172.6	2.24	2.21	.03	20.58	20.66	−.08
2	0312	121.2	8.43	8.39	.04	46.05	46.04	.01
1	0514	14.7	----	6.66	[4].18	----	27.02	[5]1.13
1	0515	40.4	----	4.41	[4].18	----	18.22	[5]1.13
1	0516	95.5	2.44	2.06	.38	29.56	28.35	1.21
2	0517	246.1	1.22	1.17	.05	23.00	22.41	.59
2	0720	213.0	2.12	2.12	0	19.72	18.79	.93
1	0721	91.8	3.17	3.11	.06	21.66	20.09	1.57
1	0722	128.6	----	----	[4].18	----	41.56	[5]1.13
2	0926	51.4	6.87	7.08	−.21	30.61	31.24	−.63
1	0927	40.4	1.80	1.85	−.05	22.64	20.49	2.15
1	0928	44.1	1.60	1.97	−.37	34.26	33.21	1.05
2	0930	51.4	2.11	2.03	.08	26.72	25.11	1.61
		Total area=1,723 acres	$\Delta\overline{M}_S$=0.059 in.			$\Delta\overline{M}_C$=0.575 in.		

[1]Type of access hole—1=river, 2=flood plain.
[2]Average measured moisture-content change in soil zone of type 2 holes.
[3]Average measured moisture-content change in capillary zone of type 2 holes.
[4]Average measured moisture-content change in soil zone of type 1 holes.
[5]Average measured moisture-content change in capillary zone of type 1 holes.

water budget. The heterogeneity of the alluvial deposits underlying the flood plain and terrace result in a wide variation in the measured moisture change between access holes as indicated by the wide range in the $\Delta\overline{M}_S$ and $\Delta\overline{M}_C$ values in table 12. A comprehensive evaluation of the measurement error of soil-moisture content was therefore undertaken to determine the reliability of these moisture-storage components.

The sampling error is the only significant error in the measurement of the change in moisture content. A bias error may exist in the measurement of the total moisture content of the soil profile because of calibration inaccuracies of the measuring equipment; but this error is one directional and nearly constant with time and thus essentially cancels when computing the change in moisture content as used in the water budget.

The sampling error in the measurement of the average change in moisture content in a reach may be attributed to the following factors:

1. Missing moisture-content data during flood periods when measurements could not be obtained at some access holes.
2. Insufficient sampling points (access holes) within the reach.
3. Improper placement of the neutron probe in the access hole or misreading the count of returning neutrons.
4. Variability in the count of returning neutrons.

The method for evaluating the total measurement error in moisture change is the same as was used in the precipitation analysis, that is, the missing-data error was first evaluated and then the rate of change in the missing-data error as the number of access holes increases was evaluated to obtain an estimate of the sampling error.

These errors were evaluated for each zone of the soil profile in reaches 1 and 2 of the flood plain and reaches 1, 2, 2a, and 3 of the terrace. These errors were then used to approximate the error in moisture change for reaches 2a and 3 of the flood plain, thus providing estimates of measurement error in moisture change for all of the areas included in the water-budget study.

The general form of the equations used in defining the measurement errors in moisture change is identical to the equations previously described for defining the precipitation error. The steps used in applying these equations are described below. All computations refer to the moisture-content data in a given zone; the same computations were made for each zone.

1. For each budget period containing a complete set of moisture-content data, compute the change in moisture content at each access hole in the reach, ΔM_{jt}, as defined by equation 29 and the average weighted change in moisture content for the reach, $\Delta\overline{M}_t$, as defined by equation 28. (Note that the array of data illustrated in table 7 for precipitation also applies to the moisture-content data by replacing P and $\overline{P}$ in the table with ΔM and $\Delta\overline{M}$, respectively.)
2. Compute $\overline{R}_j$ for each hole in the reach using equation 14 where $\overline{R}_j$ is now the average departure of moisture change in hole j, for k budget periods and

$$R_{jt} = \Delta\overline{M}_t - \Delta M_{jt}. \tag{30}$$

Moisture content data from a total of $k=53$ budget periods were used to obtain an $\overline{R}_j$ value for each of the 21 access holes in the flood plain of reach 1, and data from $k=66$ budget periods were used to obtain an $\overline{R}_j$ value for each of the nine terrace access holes in the reach. Table 13 lists the $\overline{R}_j$ values computed for each hole and each zone in the flood plain and terrace of reach 1.

3. Compute s_j for each hole in the reach using equation 16 where s_j is now the standard deviation of the

TABLE 13.—*Average departure in moisture change of each access hole* ($\overline{R}_1$) *and the standard deviation of the average departure* (s_j) *for the soil and capillary zones of the flood plain and the soil, intermediate, and capillary zones of the terrace in reach 1*

[All values are in inches]

A. Flood plain (area=1,723 acres) k=53 budget periods

Hole No.	Soil zone $\overline{R}_j$	Soil zone s_j	Capillary zone $\overline{R}_j$	Capillary zone s_j
0102	−0.089	0.495	−0.125	0.711
0103	.013	.184	−.001	.522
0104	−.003	.213	.052	.439
0105	.098	.369	.095	1.014
0106	.034	.300	−.071	.586
0308	−.008	.148	.018	.471
0309	−.001	.198	.034	.422
0310	−.045	.272	.004	.602
0311	.034	.570	.051	1.730
0312	.170	.918	.044	.982
0514	.017	.255	.021	.432
0515	.043	.491	−.038	.451
0516	−.068	.581	.064	.926
0517	−.055	.441	−.190	2.761
0720	−.026	.605	−.070	1.373
0721	.061	.353	−.038	.693
0722	.098	.565	.143	1.409
0926	.006	.620	−.029	.550
0927	.008	.199	.003	.374
0928	.047	.301	.009	.390
0930	.000	.420	.012	.527

B. Terrace (area=1,855 acres) k=66 budget periods

Hole No.	Soil zone $\overline{R}_j$	Soil zone s_j	Intermediate zone $\overline{R}_j$	Intermediate zone s_j	Capillary zone $\overline{R}_j$	Capillary zone s_j
0101	0.000	0.038	0.001	0.029	0.005	0.112
0307	.001	.060	.002	.064	−.003	.160
0312	−.002	.045	.000	.039	.007	.166
0513	.003	.043	.000	.055	.000	.208
0518	.006	.064	.001	.046	.012	.247
0719	.002	.044	.002	.033	.005	.070
0723	−.001	.032	−.001	.042	.005	.097
0724	.001	.128	−.003	.201	−.022	.323
0925	−.004	.034	−.001	.030	−.005	.110

average departure in moisture change in hole j based on k budget periods. Table 13 includes the s_j values computed for each $\bar{R}_j$ in the flood plain and terrace of reach 1.

4. Arrange the n access holes in the reach into five unique permutations (p=5), and for each permutation (v=1, . . ., 5) compute $\bar{\bar{R}}_{mv}$ for all m=1, . . ., ($n-1$) holes using equation 17 where $\bar{\bar{R}}_{mv}$ is now the mean departure in moisture change for m holes in permutation v.
5. For each of the v=1, . . ., 5 permutations, compute S_{mv} for all m=1, . . . , ($n-1$) holes using equation 18 where S_{mv} is now the standard deviation of $\bar{\bar{R}}_{mv}$ for m holes arranged in permutation v. As in the precipitation analysis, the mean and variance of the departures in moisture content were assumed to be constant within the reach. Thus, the correlation coefficient, r, included in equation 18 was approximated by equation 19. Equation 19 gives $r=-0.05$ for the 21 flood plain access holes and $r=-0.12$ for the 9 terrace access holes in reach 1.
   To test the reliability of r as an approximation of the actual correlation coefficient, $\rho_{j_1j_2}$ (see equation 20), the S_{mv} values for selected combinations of access holes in the flood plain of reach 1 were computed using $r=-0.05$ and compared with the S_{mv} values derived using the actual correlation coefficients, $\rho_{j_1j_2}$, computed from equation 20. The S_{mv} values derived by these two methods showed relatively close agreement, both for the soil zone and the capillary zone. Particularly close agreement exists between the S_{mv} values as m approaches the total number of access holes in the reach.
   No attempt was made to evaluate $\rho_{j_1j_2}$ for each combination of the nine access holes in the terrace of reach 1; however, $r=-0.12$ is considered a reasonable approximation of the actual correlation coefficients.
6. Compute the $\bar{S}_m$ of the p permutations of $\bar{\bar{R}}_{mv}$ for all m=1, ... , ($n-1$) access holes using equation 21 where $\bar{S}_m$ is now the average standard deviation of the p permutations for m access holes. Figure 7 shows a plot of the $\bar{S}_m$ values for the soil and capillary zones of the flood plain (fig. 7*A*) and for the soil, intermediate, and capillary zones of the terrace (fig. 7*B*). The curves drawn through the data points are estimates of the average error in the computed change in moisture content where data (access holes) are missing.

Table 14 lists the curve values of $\bar{S}_m$ for the flood plain and the terrace of reach 1. As with the precipitation error curves (fig. 5), a residual error also exists in the moisture-content error curves when $m=n$ holes. This residual error is attributed to differences in the s_j values of the access holes (see table 13) and the use of a constant r to approximate each $\rho_{j_1j_2}$. These curves are thus assumed to overestimate the expected standard deviation in the average moisture change.

The curves in figure 7*A* show that the error in the computed average change in moisture content for the flood plain of reach 1 does not decrease substantially beyond about 12 holes. Thus, moisture change could have been obtained for this area from about one-half the access holes actually used without a significant increase in error.

The rate of change in $\bar{S}_m$ as m increases (fig. 7) provides an estimate of the sampling error in moisture change when the change is computed from a complete set of data ($m=n$ holes). To estimate this sampling error, equations 22–25 were applied using the $\bar{S}_m$ values taken from the curves in figure 7 and listed in table 14. The

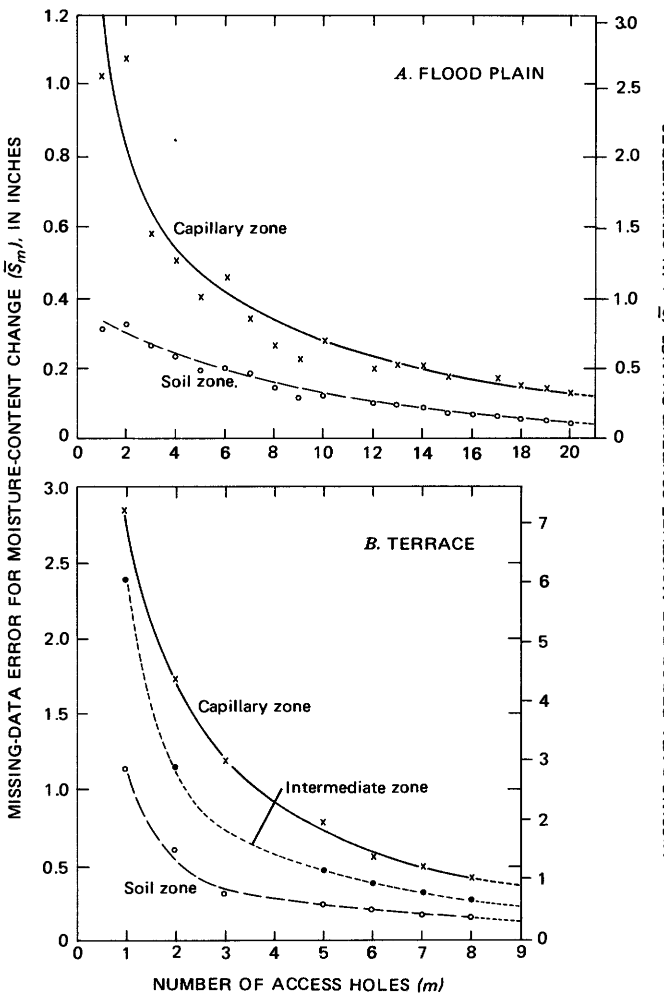

FIGURE 7.—Average relation between number of access holes in reach 1 and the missing-data error of moisture-content change per budget period for (*A*) the soil and capillary zones of the flood plain and (*B*) the soil, intermediate, and capillary zones of the terrace.

TABLE 14.—*Missing-data errors* ($\bar{S}_m$) *from curves in figure 7 for the soil and capillary zones of the flood plain and for the soil, intermediate, and capillary zones of the terrace, reach 1*
[$\bar{S}_m$ computed from p=5 permutations]

A. Flood plain $\bar{S}_m$, in inches

Number of holes	Soil zone	Capillary zone	Number of holes	Soil zone	Capillary zone
1	±0.33	±1.22	12	±0.11	±0.23
2	.30	.84	13	.10	.22
3	.27	.65	14	.09	.20
4	.24	.54	15	.08	.18
5	.22	.47	16	.07	.17
6	.20	.42	17	.07	.16
7	.17	.37	18	.06	.15
8	.16	.33	19	.05	.14
9	.14	.31	20	.04	.13
10	.13	.28	21	.04	.13
11	.12	.25			

B. Terrace $\bar{S}_m$, in inches

Number of holes	Soil zone	Intermediate zone	Capillary zone
1	±1.13	±2.39	±2.84
2	.55	1.13	1.73
3	.32	.74	1.20
4	.29	.58	.90
5	.23	.46	.71
6	.20	.38	.58
7	.17	.31	.48
8	.14	.26	.41
9	.12	.21	.36

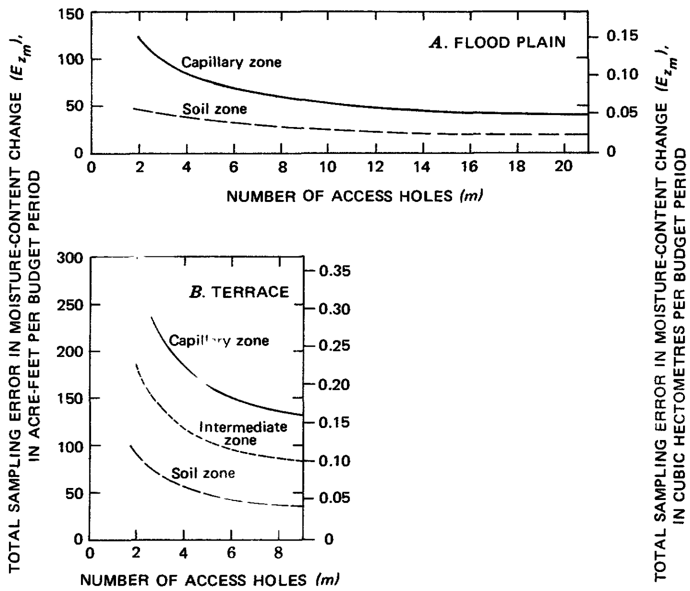

FIGURE 8.—Average relation between the number of access holes in reach 1 and the total adjusted sampling error of moisture-content change per budget period for (*A*) the soil and capillary zones of the flood plain and (*B*) the soil, intermediate, and capillary zones of the terrace.

solution of these equations give the following ∇ errors for the flood plain and terrace of reach 1:

Flood plain soil zone (n=21 holes): ∇_S =±0.118 in. (±0.300 cm)

Flood plain capillary zone (n=21 holes): ∇_C = ±0.244 in. (±0.620 cm)

Terrace capillary zone (n=9 holes): ∇_{TC} = ±0.776 in. (±1.97 cm)

These ∇ values represent the unadjusted sampling error in moisture change when $m=n$ access holes.

As with precipitation, the total adjusted sampling error in moisture change (E_{z_m}) for any given number of access holes includes both $\bar{S}_m$ and ∇. Thus, the $\bar{S}_m$ values in table 14 and the corresponding ∇ values determined above were substituted in equation 26 to define the total error curves shown in figure 8 for each zone of the flood plain and terrace in reach 1.

As indicated in table 12, soil moisture at 17 of the 21 access holes in the flood plain was measured during budget period 688–708. Entering m=17 in figure 8*A* gives the total adjusted sampling errors for the flood plain of $E_{S_{17}}$=±20 acre-ft (±0.025 hm³) in the soil zone and $E_{C_{17}}$=±42 acre-ft (±0.052 hm³) in the capillary zone. Similarly, soil moisture at eight of the nine access holes in the terrace was measured during budget period 688–708 giving, from figure 8*A*, E_{TC_8}=±138 acre-ft (±0.170 hm³) in the terrace capillary zone. Table 2 shows each of these sampling errors under their respective zone.

The procedure described above for determining the total adjusted sampling error in moisture change of each zone was also applied to the soil-moisture content data for the flood plain and terrace of reach 2 and the terraces of reaches 2a and 3. Table 15 lists these errors in moisture change computed from a complete set of data ($m=n$).

As indicated previously, moisture change in the soil and intermediate zones of the terrace are not included in the water budget (equation 1) when evaluating *ET* from the flood plain. However, the errors for these two terrace zones were independently evaluated and are included in table 15.

The estimated error in moisture change is frequently as large or larger than the measured change in moisture content for the budget period. These errors are relatively small, however, when compared with the total volume of moisture measured in the reach. For example, the total volume of moisture in the soil and capillary zones of the flood plain of reach 1 averages about 31 in. (79 cm) or 4,450 acre-ft (5.49 hm³) (p. 16). The total error in moisture change for the flood plain of reach 1, assuming no missing data, is $(E_S{}^2+E_C{}^2)^{1/2}$= $(18^2+39^2)^{1/2}$=±43 acre-ft (±0.053 hm³) from table 15. Because this error is derived from two measurements of moisture volume—one at the beginning and one at the end of the budget period—the total error for one measurement is $\sqrt{43^2/2}$=±30 acre-ft (±0.037 hm³) or only 0.7 percent of the total volume.

No independent evaluation of the sampling errors in moisture change (E_z) was made for the flood plain of

TABLE 15.—*Total adjusted sampling error in moisture change as defined with a complete set of moisture-content data* (m=n) *for each zone of the flood plain and terrace in reaches 1, 2, 2a, and 3*

Reach	Flood Plain						Terrace					
	Area (acres) A	Number of holes n	Density (acres/hole) A/n	Error (acre-ft/budget period) E_S	E_I	E_C	Area (acres) A	Number of holes n	Density (acres/hole) A/n	Error (acre-ft/budget period) E_{TS}	E_{TI}	E_{TC}
1	1,723	21	82	18	--	39	1,855	9	206	34	82	132
2	2,307	23	100	18	13	33	1,363	8	170	42	54	88
2a	1,374	13	106	[1]24	[2]18	[2]46	992	5	198	47	49	97
3	1,440	17	85	[1]20	[2]15	[3]38	602	6	100	23	23	56

[1]Estimated from equation 31. [2]Estimated from equation 32. [3]Estimated from equation 33.

reaches 2a and 3; rather, an approximation of the errors for these two areas was obtained from the previously derived errors for the flood plain of reaches 1 and 2 and the terrace of reach 3. These previously derived errors are considered to be applicable to the flood plain of reaches 2a and 3 because the areas have similar sampling densities (table 15).

To estimate the total sampling error in moisture change for the flood plain of reaches 2a and 3, the previously derived $Ez_{m|}$ values in figure 8 were first expressed in terms of sampling density by converting the m associated with each error value to the ratio A/m. The relation between Ez_m and A/m for each zone was then plotted on a semilog scale as shown in figure 9. Finally, a straight line approximating an average relation for each zone was drawn to estimate the average errors in the measured moisture-content change for the flood plain of reaches 2a and 3. Equations for these average relations are

$$E_S=-51+37\log_{10}(A/m) \tag{31}$$

$$E_I=-43+30\log_{10}(A/m) \tag{32}$$

$$E_C=-112+78\log_{10}(A/m), \tag{33}$$

where E_S, E_I, and E_C are the errors in measured moisture change in acre-feet for the soil, intermediate, and capillary zones, respectively, A is the flood-plain area of the reach in acres, and m is the number of access holes. Equations 31–33 were solved using $m=n$ to obtain the total sampling errors shown in table 15 for the flood plain of reaches 2a and 3.

Moisture-content measurements could not always be obtained at every access hole in a reach during a field visit; however, ground-water levels were generally recorded at most of the wells adjacent to the access holes. When moisture-content data from more than half of the access holes in a reach were missing for the budget period, a more reliable estimate of the change in moisture content in the capillary zone could generally be obtained from the water-level data. The relation used to obtain the capillary moisture-content change from the average water-level change is

$$\Delta\bar{M}_C=-\Delta\bar{h}S'A, \tag{34}$$

where $\Delta\bar{M}_C$ (or $\Delta\bar{M}_{TC}$) is the average moisture-content

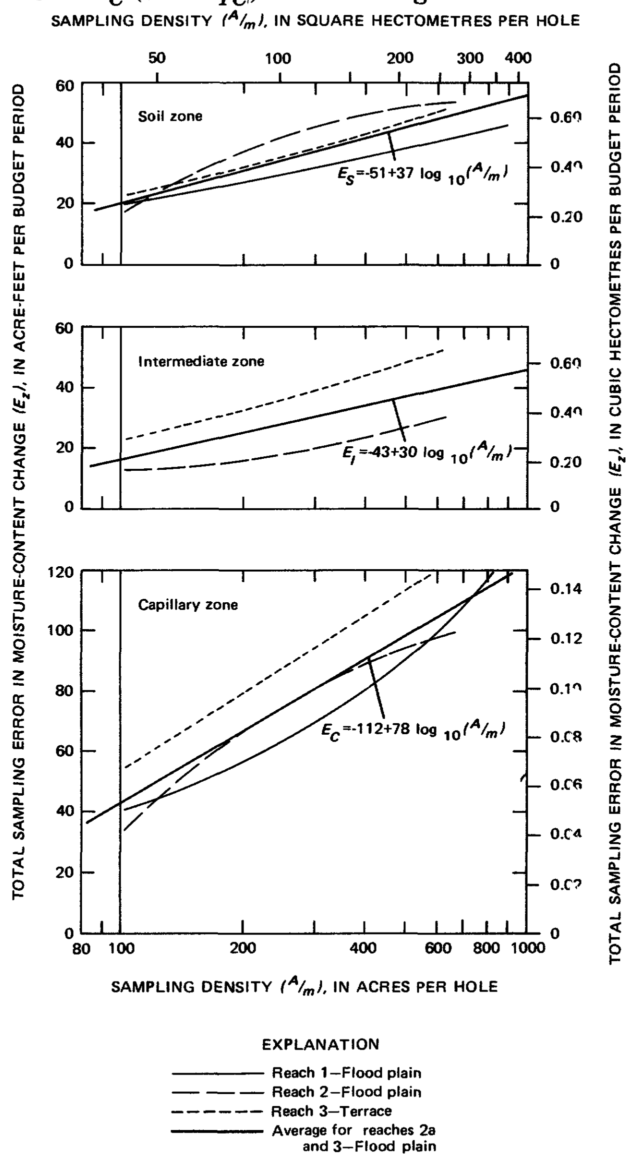

FIGURE 9.—Estimates of average relation between sampling density of access holes and total adjusted sampling error of moisture-content change per budget period for the soil, intermediate, and capillary zones of reaches 1, 2, 2a, and 3.

change in the capillary zone of the flood plain (or terrace) in acre-feet, $\Delta\bar{h}$ is the average change in the ground-water levels in the flood plain (or terrace) of the reach, in feet (positive for a rise and negative for a drop in water level), S' is the apparent specific yield of the aquifer in the zone of water-level change (dimensionless), and A is the area of the flood plain (or terrace) in acres. An average value of S' was determined for both the flood plain and terrace areas of each reach by relating the average water-level change of the area, $\Delta\bar{h}$, to the corresponding measured average moisture-content change in the capillary zone, $\Delta\bar{M}_C$ (or $\Delta\bar{M}_{TC}$), using budget periods containing a complete set of water level and moisture content data. Figure 10 shows plots of this relation for the flood plain and terrace of reach 1. The slope of the line drawn to average the data points in each relation defines the S' values used in equation 34. The $\Delta\bar{h}$ versus $\Delta\bar{M}_C$ relation for some of the areas

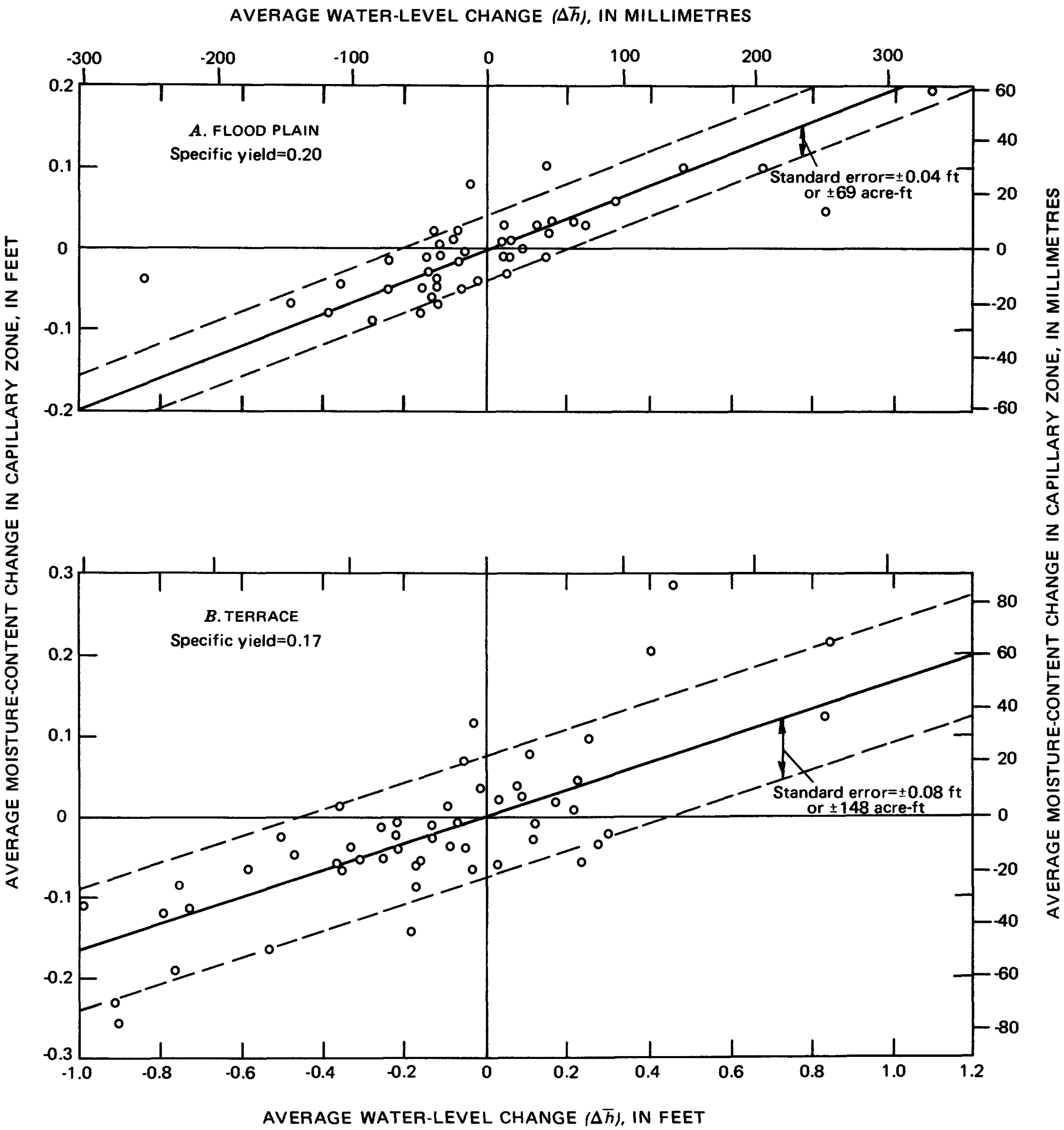

FIGURE 10.—Apparent specific yield values derived from relation between average change in water levels and average moisture-content change in capillary zones for (A) flood plain and (B) terrace of reach 1.

indicate a slight difference in S' between recharge and drainage of the aquifer; however, the variability of the data used to define the relation and the normally small changes in $\Delta\bar{h}$ did not justify defining this difference in S'. Table 16 lists the S' values and the number of budget periods, k, used to define S' for the flood plain and terrace areas of each reach.

The dotted lines paralleling the average line in each plot of figure 10 bound two-thirds of the data points, thus approximating the standard error of the measured moisture-content change in the capillary zone. These standard error values include, not only the sampling (measurement) error in moisture change for any given $\Delta\bar{h}$, but also the actual variability in moisture change due to temporal variations in ET. Thus, the error values in the figure cannot be compared directly with the sampling errors, E_C and E_{TC}, given for reach 1 in table 15.

An illustration of the application of water-level data to compute the average moisture change in the capillary zone of the flood plain of reach 1 for budget period 688–708 is given in table 17. The average weighted change in ground-water levels for the 21 wells in the flood plain is $\Delta\bar{h}=-0.386$ ft (-11.8 cm). Applying equation 34 where $S'=0.20$ (table 16) and $A=1{,}723$ acres (697 hm^2) gives $\Delta\bar{M}_C=133$ acre-feet (0.164 hm^3). This value differs considerably from the $\Delta\bar{M}_C=82$ acre-feet (0.101 hm^3) computed from the capillary moisture-content data in 17 of the 21 flood plain access holes (see table 2). But this discrepancy can be expected as indicated by the large standard error of the data points (±69 acre-feet or ±0.085 hm^3) in the $\Delta\bar{h}$ versus $\Delta\bar{M}_C$ relation of figure 10*A*.

A brief examination of the error in estimating $\Delta\bar{M}_{TC}$ from the average change in water levels was made using water-level data collected at the 10 terrace wells in reach 1. The method of analysis was identical to that used for evaluating the sampling errors in precipitation and moisture change and therefore will not be described in detail here. Water-level data collected during 20 budget periods for the 1968 water year were used in this analysis because the data provide a wide range in water-level changes. Table 18 gives the average departure in water-level change ($\bar{R}_j$) derived from equation 14 and the standard deviation of these departures (s_j) derived from equation 16 for each of the 10 wells. The missing-data error values, $\bar{S}_m$, derived from equation 21 are plotted in figure 11 and, as in the previous evaluations, indicate a residual error of $\bar{S}_m=\pm0.035$ ft (±1.07 cm) when $m=10$ wells. The application of these $\bar{S}_m$ values in equations 22–25 give an unadjusted sampling error in water-level change of $\nabla=\pm0.15$ ft (±4.6 cm). Combining the residual error and the unadjusted sampling error as in equation 2? gives an adjusted sampling error in $\Delta\bar{h}$ of $E_{\Delta\bar{h}}=\pm0.16$ ft (±4.9 cm) for $m=10$ wells. The use of water-level data to estimate moisture-content change in the water budget was seldom required, and the error in $\Delta\bar{h}$ for the other flood plain and terrace areas was not evaluated. The error $E_{\Delta\bar{h}}=\pm0.16$ ft (±4.9 cm) was therefore applied throughout the study area when using $\Delta\bar{h}$ to estimate $\Delta\bar{M}_C$ and $\Delta\bar{M}_{TC}$. Using equation 34 to express this error in terms of acre-feet of moisture change for the capillary zone of the terrace in reach 1 gives $E_{\Delta\bar{h}}=\pm50$ acre-feet (±0.062 hm^3) where S' in equation 34 is 0.17 (table 16) and A is 1,855 acres (751 ha). Table 16 lists these $E_{\Delta\bar{h}}$ values for each reach in the study area.

The total error of moisture change in the capillary zone when derived from the $\Delta\bar{h}$ vs. $\Delta\bar{M}$ relation includes not only the sampling error in $\Delta\bar{h}$ as defined above, but also the sampling error in $\Delta\bar{M}$ as defined previously for the capillary zones of the flood plain and terrace areas of

TABLE 17.—*Ground-water level changes* (Δh) *in flood plain wells of reach 1 used to compute* $\Delta\bar{M}_C$ *for budget period 688–708*

Hole No.	Area (acres)	Δh (ft)	Hole No.	Area (acres)	Δh (ft)
0102	99.2	+0.02	0514	14.7	−0.60
0103	55.1	−.57	0515	40.4	−.81
0104	33.1	−.46	0516	95.5	−.76
0105	40.4	−.21	0517	246.1	−.55
0106	29.4	−.06	0720	213.0	−.36
0308	40.4	−.33	0721	91.8	−.47
0309	36.7	−.35	0722	128.6	−.57
0310	77.1	−.31	0926	51.4	−.18
0311	172.6	−.18	0927	40.4	−.55
0312	121.2	−.06	0928	44.1	−.53
			0930	51.4	−.48

Average weighted change $\Delta\bar{h}=-0.386$ ft.
$\Delta\bar{M}_C=-(-0.386)\times0.20\times1{,}723=133$ acre-ft.

TABLE 16.—*Average apparent specific yield* (S′), *number of budget periods* (k) *used to define* S′, *total adjusted sampling error in the measurement of average water-level change* ($E_{\Delta\bar{h}}$), *and the total adjusted sampling error of moisture change in the capillary zone* ($E_{C_{\Delta\bar{h}}}$ *and* $E_{TC_{\Delta\bar{h}}}$) *when the moisture change is derived from water-level change (equation 34)*

Reach	Flood Plain k	Flood Plain S'	Flood Plain $E_{\Delta\bar{h}}$ (acre-ft)	Flood Plain $E_{C_{\Delta\bar{h}}}$ (acre-ft)	Terrace k	Terrace S'	Terrace $E_{\Delta\bar{h}}$ (acre-ft)	Terrace $E_{TC_{\Delta\bar{h}}}$ (acre-ft)
1	54	0.20	±55	±67	68	0.17	±50	±141
2	31	.26	96	101	46	.15	33	94
2a	14	.18	40	64	31	.17	27	101
3	7	.20	46	60	15	.09	9	57

TABLE 18.—*Average departure in ground-water level change* ($\bar{R}_j$) *of each terrace well in reach 1 and the standard deviation* (s_j) *of* $\bar{R}_j$ *computed from 20 budget periods of water-level data collected during the 1968 water year.*

Well No.	$\bar{R}_j$ (ft)	s_j (ft)
0101	−0.03	±0.33
0106	.02	.33
0307	−.02	.43
0312	−.04	.41
0513	.02	.18
0518	.05	.19
0719	−.01	.28
0724	.02	.36
0925	.02	.34
0930	−.03	.49

each reach. This total error may be expressed as

$$E_{C\Delta\bar{h}}=(E^2_{\Delta\bar{h}}+E^2_C)^{1/2}, \qquad (35)$$

where $E_{\Delta\bar{h}}$ is given in table 16 and E_C is the total adjusted sampling error in the capillary zone when $m=n$ (table 15). For the terrace of reach 1 this total error is $E_{TC\Delta\bar{h}}=(50^2+132^2)^{1/2}=\pm 141$ acre-feet (±0.173 hm^3). The $E_{C\Delta\bar{h}}$ and $E_{TC\Delta\bar{h}}$ error values were computed from equation 35 for the capillary zone of each reach using the $E_{\Delta\bar{h}}$ values in table 16 and the E_C and E_{TC} values in table 15. A list of these total error values is included in table 16.

BASIN-FILL INFLOW

Basin-fill inflow (G_B) to the study area is derived from deposits of low permeability which underlie the alluvium of the flood plain. This component moves vertically upward into the alluvium, and estimates of its rate of flow range from 0.07 to 1.3 ft (0.02 to 0.40 m)

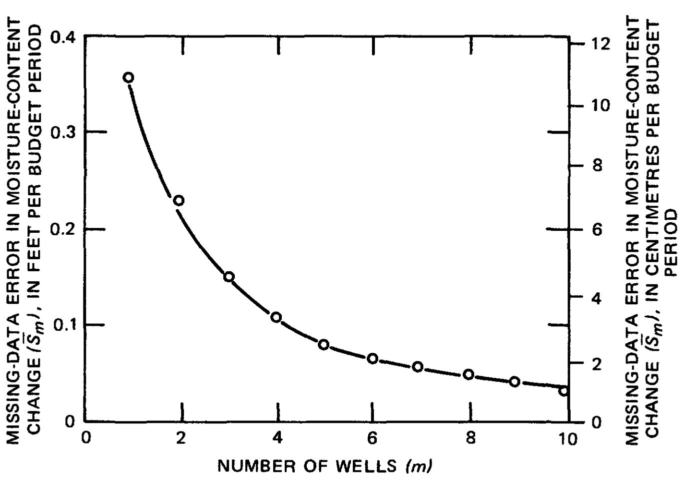

FIGURE 11.—Average relation between number of ground-water wells and missing-data error in moisture-content change per budget period for capillary zone of terrace in reach 1.

per year (Hanson, 1972, p. F27). If all basin-fill inflow surfaced as evapotranspiration, these values would represent an *ET* rate between 15 and 265 acre-ft (0.018 and 0.326 hm^3) per 21 days for reach 1. The constraints and limitations associated with the methods used to derive these G_B values make these estimates questionable.

A subsequent analysis of the moisture movement in the capillary zone of the deep terrace wells of reach 1 did, however, provide a better indication of basin-fill inflow to the study area. In the analysis, only moisture data from the winter months, when *ET* is minimal and cross-valley ground-water slopes are negligible, was evaluated. The results of this study indicate that the basin-fill inflow is about 0.3 ft (0.09 m) per year per unit area of flood plain or G_B=62 acre-ft (0.076 hm^3) per 21 days for reach 1. This value is believed to be a more reasonable estimate of the true rate of basin-fill inflow.

The basin-fill inflow was assumed to remain constant throughout the year, unaffected by seasonal variations in barometric pressure, temperature, or ground-water levels. Because G_B is considered time invariant, its sampling error is zero as indicated under item 9 of table 2.

No evaluation of the bias in the estimate of basin-fill inflow was possible. Thus, it is assumed that the estimate is 100 percent in error as indicated by the bias error value of $E_{G_B}=\pm 62$ acre-ft (±0.076 hm^3) under item 9 in table 2. This bias may be significant when evaluating *ET* for a given budget period, but the bias cancels when computing the change in *ET* from before-clearing and after-clearing *ET* data.

DOWNVALLEY GROUND-WATER FLOW

Ground-water movement downvalley through the upstream and downstream ends of each reach was calculated from

$$G=iTWD, \qquad (36)$$

where

- G = downvalley ground-water flow through the alluvium in acre-feet per budget period,
- i = average downvalley gradient of the ground-water level during the budget period through the upstream or downstream end of the reach,
- T = transmissivity of the alluvium in acre-feet per day per foot,
- W = width of saturated alluvium at the upstream or downstream end of the reach, in feet, and
- D = number of days in the budget period.

The transmissivity, T, of the alluvium was assumed to be constant throughout the study area and was determined by Hanson (1972, p. F27) to be 28,000 ft^3 per day per foot or 0.644 acre-ft per day per foot (2,600 m^3 per day per metre). The width of saturated alluvium, W, was determined by measuring the distance between the points of contact of the alluvium with the basin fill at the water table on each side of the flood plain. The downvalley slope, i, was computed from the average ground-water levels for the budget period measured at the river wells and flood-plain wells on and adjacent to the cross sections at the ends of the reach. For example, the slope through the upstream end of reach 1 (cross section 1 in figure 1) was computed from the average water levels measured in the river wells and flood-plain wells at cross sections 1 and 3. Similarly, the slope through the downstream end of the reach (cross section 9) was computed from the average water levels in the wells at adjacent cross sections 7 and 11. The calculations used to obtain G_I and G_O in table 2 are

$$G_I = 0.00158 \times 0.644 \times 5{,}800 \times 21 = 124 \text{ acre-ft/21 days } (0.153 \text{ hm}^3\text{/21 days})$$

$$G_O = 0.00148 \times 0.644 \times 5{,}600 \times 21 = 112 \text{ acre-ft/21 days } (0.138 \text{ hm}^3\text{/21 days}).$$

The sampling error associated with the G_I and G_O components is dependent only on the sampling error of i in equation 36 because i is the only factor in the equation which is measured for each budget period. The factors T and W do not have a sampling error, because they are considered constant with time (actually T and W may vary slightly with large changes in water level) and T is assumed to be constant throughout all reaches.

Seasonal variations in downvalley slope through most cross sections are generally less than 5 percent during periods of minimum ground-water level fluctuations. Most of this variability reflects changes in the ground-water level caused by precipitation, changes in the river stage and seasonal variations in ET. As a result, no detailed evaluation of the sampling error in slope was possible. An approximation of this error was obtained, however, by examining the variability in the measured downvalley slope during the winter months when ET is negligible and the ground-water level remains relatively stable. Figure 12 shows the average downvalley slopes through cross section 9 for the winter months (November through February) of water years 1964, 1965, 1967, 1969, and 1970. Variability about the general trends in these slopes suggests that the error in i for any given budget period is probably less than $\pm 0.005 \times 10^{-3}$ or ± 0.3 percent of the slope. An error of ± 0.3 percent gives an average error in the downvalley ground-water movement of ± 0.4 acre-ft (± 0.0005 hm^3) per 21 days for the ground-water inflow and outflow components in table 2. Because this sampling error is so small, it is considered zero as shown under the G_I and G_O components in table 2.

Any bias error in the ground-water components, G_I and G_O, is attributed solely to W and T in equation 36. The bias error in G_I and G_O resulting from an inaccurate determination of W was estimated to be ± 200 ft (± 60 m) or 4 percent of W. This estimate is probably high and is believed to be closer to ± 100 ft (± 30 m). Considering that the downvalley ground-water inflow to reach 1 is $G_I = 124$ acre-ft (0.153 hm^3) for budget period 688–708, the error in W of 4 percent gives a bias error in G_I of ± 5 acre-ft (± 0.006 hm^3).

The bias in G_I and G_O attributed to using a constant average T for all reaches was estimated by assuming that the spatial variability in downvalley slopes, not explained by differences in flood-plain width, was a direct measure of the variability in T. The average downvalley slopes between the cross sections at the ends of each reach are plotted in figure 13 against the width

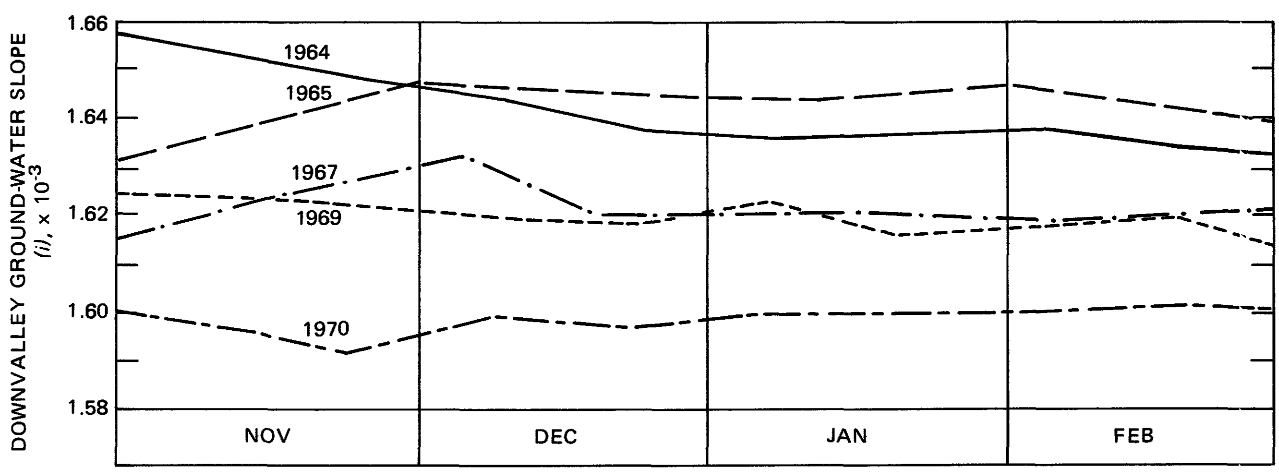

FIGURE 12.—Average downvalley ground-water slopes through cross section 9 for winter months of the 1964, 1965, 1967, 1969, and 1970 water years.

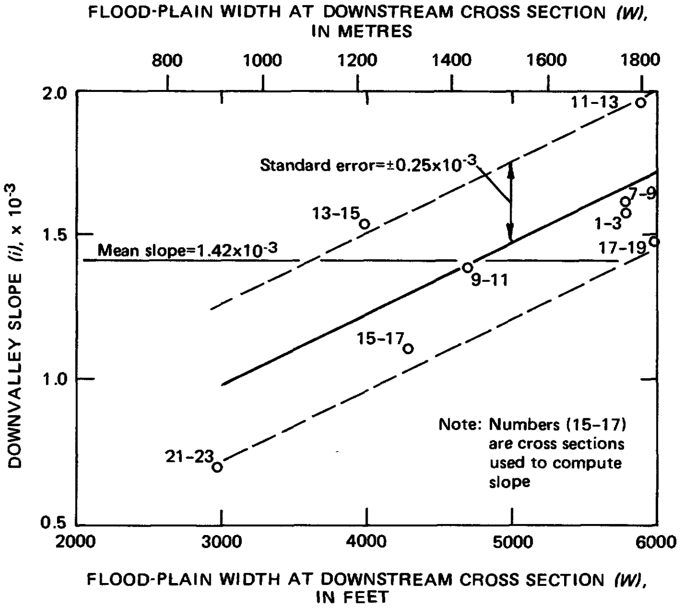

FIGURE 13.—Relation between average downvalley ground-water slope through the upstream and downstream ends of each reach in the study area and width of the flood plain at the downstream cross section used to compute the slope.

of the downstream cross section used in the slope computation. The plot indicates that the spatial variability in slope, after the effect of differences in cross-section width is removed, is $\pm 0.25\times 10^{-3}$ or about 18 percent of the mean slope of 1.42×10^{-3}. Assuming that this variability reflects the spatial variation in T, the bias error in the ground-water components attributed to using a constant T is 18 percent of the value of the component. For the ground-water inflow component $G_I=124$ acre-ft (0.153 hm^3), this bias error is $\pm 0.18\times 124=\pm 22$ acre-ft ($\pm$0.027 hm^3). The total bias in G_I due to the bias errors in W and T is then $E_{G_I}=\sqrt{5^2+22^2}=23$ acre-ft ($\pm$0.028 hm^3). Similar computations show that the total bias in G_O is $E_{G_O}=\pm 21$ acre-ft ($\pm$0.026 hm^3) for budget period 688–708. These bias errors are shown under the G_I and G_O components of table 2.

Because the bias error is computed as a fraction of G, the error will approach zero as G approaches zero. It was assumed in this analysis, however, that a bias error always exists because of the uncertainty in the measurement of a zero slope. Thus, a bias error of $\pm$0.8 acre-ft ($\pm$0.001 hm^3) per day, which corresponds to the average measurement error in slope (fig. 13), was arbitrarily set as the minimum total bias error.

COMPUTATION OF *ET* AND TOTAL ERROR IN *ET*

The total ET for a budget period is obtained by algebraically summing the 12 components of the water budget as expressed in equation 1. For the example budget period in table 2, this summation gives $ET=513$ acre-ft (0.633 hm^3) per 21 days.

The components of the water budget in reach 1 for each budget period of the 1964 water year (table 1) have been grouped into four principal sources of water (fig. 14) to illustrate the relative significance of each source. The algebraic summation of the bar graph values for any given budget period in the figure gives ET in acre-ft per 14 days. The graph of surface water sources does not indicate the amount of discharge in the Gila River and its tributaries but rather the loss (or gain) of flow through the reach during the budget period. The primary components in the surface water sources are the Gila River inflow (Q_I) and outflow (Q_O). The channel storage (ΔC) and the tributary inflow (Q_I) components are generally only a small part of the total surface-water source (see also table 1). The graph indicates that surface water is the most significant source contributing to ET during the winter and late summer months of the 1964 water year.

The soil-moisture components ($\Delta\overline{M}_S$, $\Delta\overline{M}_C$, and $\Delta\overline{M}_{TC}$) are the most significant sources of water in the water budget during May and June, when the contribution from surface water is minimal and ET rates are approaching a maximum. The precipitation ($\overline{P}$) and ground-water sources (G_B, G_I, and G_O) are relatively insignificant during most of the year, with the ground-water components contributing a nearly constant 50 acre-ft (0.062 hm^3) per 14 days to ET throughout the year.

This report has shown that nine of the components contain significant sampling errors and three of the components contain significant bias errors. Even though some of these components are interrelated, the measurement of each component is based on an independent observation. Thus, the estimate of the total error in ET is treated as an expected value of the error variance of each term. The total sampling error in ET may therefore be obtained from

$$E_{ET_s}=\left(E^2_{Q_I}+E^2_{Q_O}+E^2_{Q_T}+E^2_{\Delta C}+E^2_{\overline{P}}+E^2_{S}+E^2_{I}+E^2_{C}+E^2_{TC}\right)^{1/2}, \quad (37)$$

where E_{ET_s} is the total sampling error in ET and the error terms on the right side of the equation are as defined previously. For budget period 688–708, this error is $E_{ET_s}=\pm 344$ acre-ft ($\pm$0.412 hm^3).

The total bias error in ET may be obtained from

$$E_{ET_b}=\left(E^2_{GB}+E^2_{GI}+E^2_{GO}\right)^{1/2}, \quad (38)$$

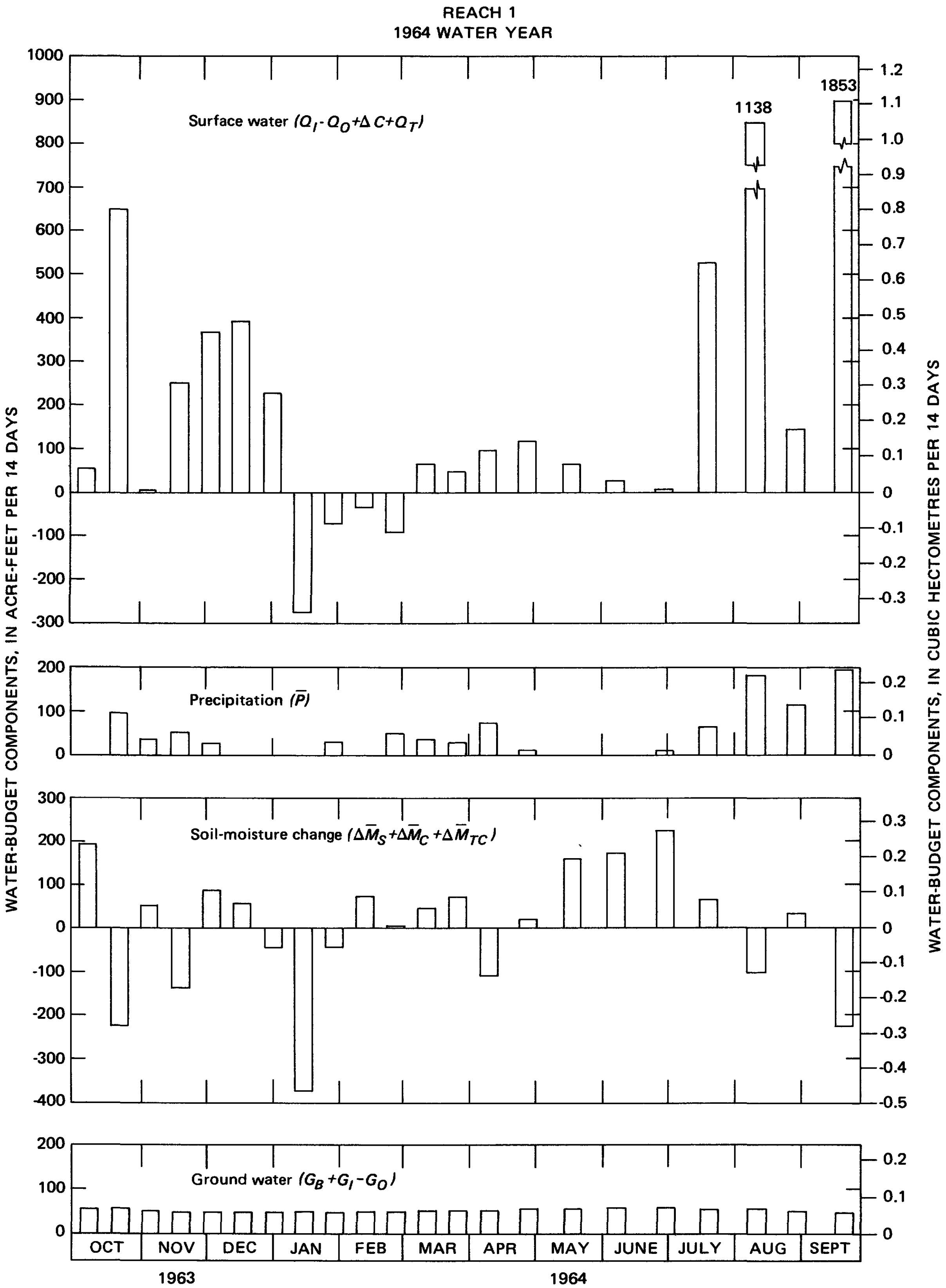

FIGURE 14.—Graph showing sources of water contributing to the total ET per budget period for the 1964 water year, reach 1.

where E_{ET_b} is the expected bias error in ET and the error terms on the right side of the equation are as defined previously. As in equation 37, equation 38 also assumes that the bias errors of each component are independent and unknown as to direction. For budget period 688–708, $E_{ET_b}=\pm 69$ acre-ft (± 0.085 hm^3).

The total measurement error in ET attributed to both the bias and the sampling errors is

$$E_{ET}=(E^2_{ET_s}+E^2_{ET_b})^{1/2}. \qquad (39)$$

For budget period 688–708, this total error is $E_{ET}=\sqrt{(334)^2+(69)^2}=\pm 341$ acre-ft (± 0.420 hm^3), which is 66 percent of the computed ET.

Figure 15*A* shows the ET values computed for each budget period in reach 1 during the 1964 water year and the errors associated with each ET value. The brackets bounding these values define the total measurement error (E_{ET}) in ET. Included within these brackets are bars indicating the error in ET attributed to the streamflow components Q_I and Q_O and the error attributed to the soil-moisture change components $\Delta\overline{M}_S$, $\Delta\overline{M}_C$, and $\Delta\overline{M}_{TC}$. The hydrograph of the Gila River at cross section 9 in figure 15*B* shows that the magnitude of the streamflow errors is directly related to the discharge, with the largest errors occurring during periods of highest discharge.

The seasonal trend in ET is indicated in figure 15*A* by the average potential ET curve. This curve was determined by using the average daily temperatures and the number of daylight hours for the study area (Blaney and Criddle, 1962) and by assuming that sufficient moisture is always available to satisfy the demand for vaporization; therefore, the curve approximates the upper limit of ET throughout the year. Actual ET may exceed this potential curve, however, because the curve is only an estimate of the potential rate and does not account for all the factors controlling ET. Most of the water-budget ET values in figure 15*A* which exceed this curve contain measurement errors that fall well below the curve, suggesting that the measurement errors are, in most instances, at least as large as the expected standard error in ET. Also, those ET values with the lowest measurement error follow, in general, the trend defined by the potential curve. Some ET values in the water budget are negative, but their measurement errors are generally large and extend into the positive ET range. In a few instances the computed measurement errors do not explain large negative ET values (as in January in fig. 15*A*) or unrealistically high ET values. These outliers generally occur during periods of high streamflow and are assumed to reflect large unmeasured changes in the stage-discharge relations which are not fully accounted for in the streamflow error analysis. They may also reflect unknown quantities of surface water moving into or out of depression storage as described on page 9.

One of the most important points realized from figures 14 and 15 is that the total measurement error in ET is dependent on the volume of water moving through the reach and not the magnitude of ET. This is emphasized in figure 15*A* by the nearly constant error in moisture change for each ET value reflecting not the large variation in moisture change shown in figure 14 but rather the total volume of soil moisture measured in the reach which fluctuates relatively little with time.

None of the 12 water-budget components has both a sampling error and a bias error. This circumstance is unique to this study area and should not be expected to occur in other areas having the same type of components, particularly if the components are of different hydrologic significance. For example, in areas where ground-water movement is comparatively large, the sampling error may also be large and contribute significantly to the total measurement error in ET. A bias error in any one of the water-budget components may also become significant if the frequency of data collection or the sampling density do not adequately describe the temporal and spatial changes in the component.

During the 9-year study, a total of 416 ET values were computed from reaches 1, 2, 2a, and 3. Table 19 lists these ET values for each budget period and each reach and gives the sampling error (E_{ET_s}) and total error (E_{ET}) for each ET value. About 60 percent of the ET values have a measurement error which exceeds the ET value. However, as noted previously, the assumptions and criteria used in this analysis give measurement errors which, in most instances, would be expected to exceed the standard error of estimate.

COMPUTATION OF $\Delta\overline{ET}$ AND ERROR IN $\Delta\overline{ET}$

One of the principle objectives of the Gila River Phreatophyte Project is to determine the salvage of water as defined by the change in evapotranspiration following removal of phreatophytes from the flood plain. The average change in evapotranspiration derived from ET data obtained before and after clearing for the June–July period is presented in this section to illustrate both the magnitude and the measurement variability of this ET change.

The average change ($\Delta\overline{ET}$) is

$$\Delta\overline{ET}=\overline{ET}_B-\overline{ET}_A, \qquad (40)$$

where $\overline{ET}_B$ and $\overline{ET}_A$ are the average evapotranspiration rates for given periods of time before and after

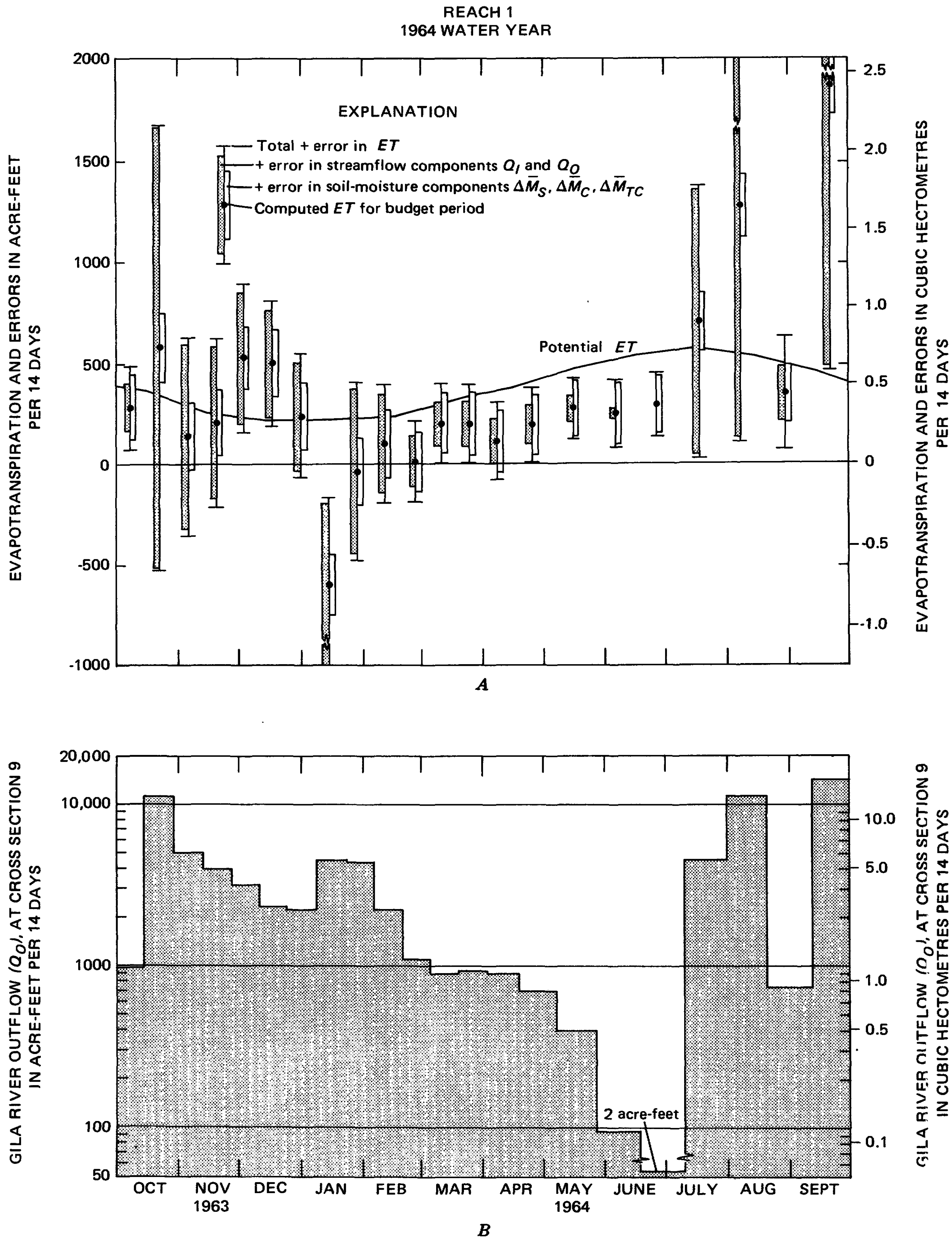

FIGURE 15.—Graphs of (A) ET and corresponding errors in ET per budget period and (B) outflow of Gila River at cross section 9 for the 1964 water year, reach 1.

TABLE 19.—*Evapotranspiration* (ET) *and total measurement error of evapotranspiration* (E_{ET}) *for each budget period during water years 1963–71—reaches 1, 2, 2a, and 3*

[All values are in acre-feet per budget period]

Reach 1

1963 water year

Budget period	Number of days	*ET*	E_{ET}
03-19-63	14	521	±478
04-02-63	14	30	249
04-16-63	14	129	248
04-30-63	14	228	215
05-14-63	14	−171	194
05-28-63	14	312	193
06-11-63	14	389	178
06-25-63	14	220	158
07-09-63	14	306	159
07-23-63	14	254	165
08-06-63	14	446	598
08-20-63	14	851	490
09-03-63	14	4122	1997
09-17-63	14	2556	1336

1964 water year

Budget Period	Number of days	*ET*	E_{ET}
10-01-63	14	728	±727
10-15-63	14	293	210
10-29-63	14	582	1106
11-12-63	14	137	496
11-26-63	14	209	419
12-10-63	14	525	364
12-24-63	14	494	315
01-07-64	14	230	311
01-21-64	14	−604	438
02-04-64	14	−40	445
02-18-64	14	97	296
03-03-64	14	9	200
03-17-64	14	195	192
03-31-64	14	196	197
04-14-64	14	112	195
05-04-64	20	269	200
05-25-64	21	410	184
06-15-64	21	372	172
07-06-64	21	424	161
07-27-64	21	1056	819
08-17-64	21	1903	1427
09-07-64	21	513	341
09-28-64	21	2799	1732

1965 water year

Budget period	Number of days	*ET*	E_{ET}
10-19-64	21	229	±409
11-09-64	21	132	177
11-30-64	21	132	262
12-21-64	21	113	275
01-11-65	21	42	408
02-01-65	21	−193	776
02-22-65	21	465	757
03-15-65	21	247	655
04-05-65	21	33	545
04-26-65	21	378	470
05-17-65	21	82	335
06-07-65	21	407	187
06-28-65	21	448	159
07-19-65	21	469	164
08-09-65	21	1979	1294
08-30-65	21	265	600
09-20-65	21	1224	1247

1966 water year

Budget period	Number of days	*ET*	E_{ET}
10-11-65	21	436	±206
11-01-65	21	302	192
11-22-65	21	265	221
01-24-66	63		
02-14-66	21	−228	1467
03-07-66	21	1702	1464
03-28-66	21		
04-18-66	21		
05-09-66	21	999	1044
05-30-66	21	683	456
06-20-66	21	671	228
07-11-66	21	738	188
08-01-66	21	405	428
08-22-66	21	911	501
09-12-66	21	564	633

1967 water year

Budget period	Number of days	*ET*	E_{ET}
10-03-66	21	366	±1096
10-24-66	21	109	264
11-14-66	21	350	258
12-05-66	21	263	325
12-19-66	14	110	215
01-16-67	28	44	286
02-06-67	21	144	363
02-27-67	21	105	206
03-20-67	21	−273	206
04-10-67	21	199	190
05-01-67	21	75	181
05-22-67	21	221	169
06-12-67	21	327	162
07-03-67	21	200	166
07-24-67	21	413	1048
08-14-67	21		
09-04-67	21		
09-25-67	21	738	810

1968 water year

Budget period	Number of days	*ET*	E_{ET}
10-16-67	21	446	±805
11-06-67	21	159	306
11-27-67	21	212	302
12-18-67	21	397	464
01-08-68	21		
01-29-68	21	1767	2181
02-19-68	21		
03-11-68	21		
04-01-68	21	1228	3589
04-22-68	21		
05-13-68	21	−999	1649
06-03-68	21	752	882
06-24-68	21	403	314
07-08-68	14	351	211
07-22-68	14	25	173
08-05-68	14	254	426
08-19-68	14	−55	998
09-02-68	14	55	632
09-16-68	14	348	317
09-30-68	14	90	178

1969 water year

Budget period	Number of days	*ET*	E_{ET}
10-14-68	14	−42	±188
10-28-68	14	161	175
11-11-68	14	97	225
11-25-68	14	83	384
12-09-68	14	−31	426
12-23-68	14	244	327
01-06-69	14	−235	501
01-20-69	14	317	535
02-03-69	14	78	663
02-17-69	14	199	573
03-03-69	14	105	350
03-17-69	14	99	251
03-31-69	14	0	225
04-14-69	14	56	234
04-28-69	14	96	213
05-12-69	14	98	212
05-26-69	14	200	191
06-09-69	14	138	184
06-23-69	14	192	176
07-07-69	14	228	176
07-21-69	14	183	261
08-04-69	14	89	183
08-18-69	14	154	223
09-01-69	14	−17	193
09-15-69	14	−233	702
09-29-69	14	183	279

1970 water year

Budget period	Number of days	*ET*	E_{ET}
10-13-69	14	208	±169
10-27-69	14	207	221
11-10-69	14	−67	221
11-24-69	14	112	292
12-08-69	14	86	346
12-22-69	14	−87	360
01-05-70	14	106	227
01-19-70	14	117	247
02-02-70	14	98	215
02-16-70	14	40	208
03-09-70	21	127	435
03-23-70	14	81	242
04-06-70	14	135	215
04-20-70	14	18	207
05-04-70	14	19	204
05-18-70	14	72	186
06-01-70	14	23	177
06-15-70	14	136	170
06-29-70	14	269	166
07-13-70	14	154	164
07-27-70	14	209	176
08-10-70	14	153	282
08-24-70	14	183	235
09-07-70	14	262	168
09-21-70	14	193	196

1971 water year

Budget period	Number of days	*ET*	E_{ET}
10-05-70	14	568	±750
10-19-70	14	113	306
11-02-70	14	97	177
11-16-70	14	41	187
11-30-70	14	14	179
12-14-70	14	55	184
12-28-70	14	5	184
01-11-71	14	126	328
01-25-71	14	−40	376
02-08-71	14	91	351
02-22-71	14	356	326
03-08-71	14	−8	274
03-22-71	14	−88	212
04-05-71	14	27	186
04-19-71	14	74	184
05-03-71	14	−20	180
05-17-71	14	41	173
05-31-71	14	82	169
06-14-71	14	93	165
06-28-71	14	106	163
07-12-71	14	203	165
07-26-71	14	192	177
08-09-71	14	−728	462
08-23-71	14		
09-06-71	14	518	626
09-20-71	14	−183	463

TABLE 19.—*Evapotranspiration* (ET) *and total measurement error of evapotranspiration* (E_{ET}) *for each budget period during water years 1963–71—reaches 1, 2, 2a, and 3*—Continued

Reach 2

1963 water year

Budget period	Number of days	*ET*	E_{ET}
07-09-63	14	427	±154
07-23-63	14	573	130
08-06-63	14	−195	566
08-20-63	14	−455	468
09-03-63	14	329	1832
09-17-63	14	−755	1275

1964 water year

Budget period	Number of days	*ET*	E_{ET}
10-08-63	21	−293	±749
10-22-63	14	111	967
11-05-63	14	535	543
11-19-63	14	437	457
12-03-63	14	−53	360
12-17-63	14	−246	322
12-31-63	14	97	219
01-14-64	14	27	363
01-28-64	14	167	453
02-11-64	14	92	368
02-25-64	14	104	222
03-10-64	14	302	191
03-24-64	14	103	185
04-07-64	14	246	159
04-27-64	20	143	168
05-18-64	21	387	153
06-08-64	21	728	135
06-29-64	21	789	141
07-20-64	21	1058	647
08-10-64	21	1068	1257
08-31-64	21	537	763
09-21-64	21	439	791

1965 water year

Budget period	Number of days	*ET*	E_{ET}
10-12-64	21	−409	±1495
11-02-64	21	363	177
11-23-64	21	85	202
12-14-64	21	145	269
01-04-65	21	197	227
01-25-65	21	283	712
02-15-65	21	293	728
03-08-65	21	323	680
03-29-65	21	−391	564
04-19-65	21	−65	491
05-10-65	21	793	368
05-31-65	21	616	171
06-21-65	21	563	128
07-12-65	21	832	129
08-02-65	21	1195	946
08-23-65	21	842	821
09-13-65	21	1183	1184

1966 water year

Budget period	Number of days	*ET*	E_{ET}
10-04-65	21	763	±346
10-25-65	21	562	148
11-15-65	21	250	181
12-06-65	21	253	290
02-07-66	60		
02-28-66	21	−838	1549
03-21-66	21		
04-11-66	21		
05-02-66	21		
05-23-66	21		
06-13-66	21		
07-04-66	21		
07-25-66	21		
08-15-66	21	604	221
09-05-66	21	460	738
09-26-66	21	−98	1117

1967 water year

Budget period	Number of days	*ET*	E_{ET}
10-17-66	21	795	±284
11-07-66	21	323	206
11-28-66	21	64	314
12-12-66	14	132	210
01-09-67	28	250	253
01-30-67	21	−48	355
02-20-67	21	54	214
03-13-67	21	225	218
03-27-67	14	17	184
04-17-67	21	285	163
05-08-67	21	271	146
05-29-67	21	720	135
06-19-67	21	739	131
07-10-67	21	974	170
07-31-67	21	531	991
08-21-67	21		
09-11-67	21	464	664

1968 water year

Budget period	Number of days	*ET*	E_{ET}
10-02-67	21	−212	±1070
10-23-67	21	219	421
11-13-67	21	403	291
12-04-67	21	318	323
12-25-67	21		
01-15-68	21		
02-05-68	21		
02-26-68	21		
03-18-68	21		
04-08-68	21		
04-29-68	21		
05-20-68	21		
06-10-68	21		
07-01-68	21		
07-15-68	14		
07-29-68	14		
08-12-68	14		
08-26-68	14		
09-09-68	14		
09-23-68	14		

1969 water year

Budget period	Number of days	*ET*	E_{ET}
10-07-68	14		
10-21-68	14		
11-04-68	14		
11-18-68	14		
12-02-68	14		
12-16-68	14		
12-30-68	14		
01-13-69	14		
01-27-69	14		
02-10-69	14		
02-24-69	14		
03-10-69	14		
03-24-69	14		
04-07-69	14		
04-21-69	14		
05-05-69	14		
05-19-69	14		
06-02-69	14	289	±139
06-16-69	14	527	128
06-30-69	14	360	119
07-14-69	14	510	125
07-28-69	14	1026	183
08-11-69	14	721	125
08-25-69	14	576	155
09-08-69	14		
09-22-69	14		

1970 water year

Budget period	Number of days	*ET*	E_{ET}
10-06-69	14	194	±128
10-20-69	14	419	122
11-03-69	14	194	189
11-17-69	14	344	183
12-01-69	14	24	312
12-15-69	14	474	352
12-29-69	14	220	243
01-12-70	14	1	228
01-26-70	14	28	198
02-09-70	14	−62	179
02-23-70	14	179	174
03-16-70	21	306	436
03-30-70	14	165	202
04-13-70	14	270	179
04-27-70	14	143	179
05-11-70	14	257	162
05-25-70	14	272	145
06-08-70	14	314	133
06-22-70	14	520	124
07-06-70	14	583	122
07-20-70	14	587	125
08-03-70	14	479	136
08-17-70	14	558	280
08-31-70	14	505	154
09-14-70	14	507	137
09-28-70	14	275	156

1971 water year

Budget period	Number of days	*ET*	E_{ET}
10-12-70	14		
10-26-70	14	187	±143
11-09-70	14	77	147
11-23-70	14	177	140
12-07-70	14	122	146
12-21-70	14	110	148
01-04-71	14	177	199
01-18-71	14	−91	353
02-01-71	14	31	351
02-15-71	14	−134	318
03-01-71	14	260	270
03-15-71	14	53	226
03-29-71	14	8	161
04-12-71	14	170	146
04-26-71	14	268	143
05-10-71	14	125	143
05-24-71	14	127	135
06-07-71	14	205	125
06-21-71	14	188	122
07-05-71	14	144	122
07-19-71	14	245	154
08-02-71	14	340	429
08-16-71	14		
08-30-71	14		
09-13-71	14		
09-27-71	14		

TABLE 19.—*Evapotranspiration* (ET) *and total measurement error of evapotranspiration* (E_{ET}) *for each budget period during water years 1963–71—reaches 1, 2, 2a, and 3*—Continued

Reach 2a

1966 water year

Budget period	Number of days	*ET*	E_{ET}
10-04-65	21		
10-25-65	21		
11-15-65	21		
12-06-65	21		
02-07-66	63		
02-28-66	21		
03-21-66	21		
04-11-66	21		
05-02-66	21		
05-23-66	21		
06-13-66	21		
07-04-66	21	656	±178
07-25-66	21	663	154
08-15-66	21	203	223
09-05-66	21		
09-26-66	21		

1967 water year

Budget period	Number of days	*ET*	E_{ET}
10-17-66	21	−621	±349
11-07-66	21	228	205
11-28-66	21	180	312
12-12-66	14	198	209
01-09-67	28	92	255
01-30-67	21	−125	361
02-20-67	21	−78	219
03-13-67	21	102	212
03-27-67	14	28	177
04-17-67	21	195	167
05-08-67	21	86	149
05-29-67	21	388	137
06-19-67	21	343	139
07-10-67	21	323	183
07-31-67	21		
08-21-67	21		
09-11-67	21		

1968 water year

Budget period	Number of days	*ET*	E_{ET}
10-02-67	21		
10-23-67	21		
11-13-67	21	314	±290
12-04-67	21	188	325
12-25-67	21		
01-15-68	21		
02-05-68	21		
02-26-68	21		
03-18-68	21		
04-08-68	21		
04-29-68	21		
05-20-68	21		
06-10-68	21		
07-01-68	21		
07-15-68	14		
07-29-68	14	151	177
08-12-68	14		
08-26-68	14		
09-09-68	14		
09-23-68	14	328	173

1969 water year

Budget period	Number of days	*ET*	E_{ET}
10-07-68	14	379	±139
10-21-68	14	232	144
11-04-68	14	27	170
11-18-68	14	106	255
12-02-68	14	230	416
12-16-68	14	−31	357
12-30-68	14	42	418
01-13-69	14	−325	487
01-27-69	14	−111	567
02-10-69	14	−706	683
02-24-69	14	47	452
03-10-69	14	55	251
03-24-69	14	186	207
04-07-69	14	102	193
04-21-69	14	141	193
05-05-69	14	223	169
05-19-69	14	45	169
06-02-69	14	68	142
06-16-69	14	218	130
06-30-69	14	146	128
07-14-69	14	187	126
07-28-69	14	302	165
08-11-69	14	301	132
08-25-69	14	350	174
09-08-69	14		
09-22-69	14		

1970 water year

Budget period	Number of days	*ET*	E_{ET}
10-06-69	14	34	±153
10-20-69	14	249	128
11-03-69	14	45	198
11-17-69	14	269	186
12-01-69	14	−114	321
12-15-69	14	262	356
12-29-69	14	35	248
01-12-70	14	−9	228
01-26-70	14	−36	199
02-09-70	14	−131	178
02-23-70	14	132	171
03-16-70	21	327	425
03-30-70	14	125	198
04-13-70	14	202	177
04-27-70	14	160	174
05-11-70	14	161	162
05-25-70	14	66	145
06-08-70	14	111	134
06-22-70	14	153	125
07-06-70	14	229	122
07-20-70	14	180	120
08-03-70	14	151	135
08-17-70	14	169	290
08-31-70	14	232	149
09-14-70	14	262	130
09-28-70	14	40	159

1971 water year

Budget period	Number of days	*ET*	E_{ET}
10-12-70	14		
10-26-70	14		
11-09-70	14		
11-23-70	14	55	±141
12-07-70	14	73	145
12-21-70	14	32	147
01-04-71	14	138	199
01-18-71	14	−11	351
02-01-71	14	216	344
02-15-71	14	−237	321
03-01-71	14	123	272
03-15-71	14	10	228
03-29-71	14	−7	159
04-12-71	14	95	145
04-26-71	14	121	143
05-10-71	14	77	137
05-24-71	14	112	128
06-07-71	14	92	125
06-21-71	14	109	121
07-05-71	14	77	120
07-19-71	14	155	131
08-02-71	14		
08-16-71	14		
08-30-71	14		
09-13-71	14	219	365
09-27-71	14		

Reach 3

1964 water year

Budget period	Number of days	*ET*	E_{ET}
10-08-63	21		
10-22-63	14		
11-05-63	14		
11-19-63	14		
12-03-63	14		
12-17-63	14		
12-31-63	14		
01-14-64	14		
01-28-64	14	25	±446
02-11-64	14	36	361
02-25-64	14	−27	209
03-10-64	14	47	168
03-24-64	14	−34	170
04-07-64	14	70	165
05-11-64	34	246	190
06-01-64	21	433	95
06-22-64	21	541	94
07-13-64	21	580	90
08-03-64	21		
08-24-64	21		
09-14-64	21		

1965 water year

Budget period	Number of days	*ET*	E_{ET}
10-05-64	21		
10-26-64	21		
11-16-64	21	239	±114
12-07-64	21	46	246
12-28-64	21	108	209
01-18-65	21		
02-08-65	21		
03-01-65	21		
03-22-65	21		
04-12-65	21		
05-03-65	21	400	390
05-24-65	21	374	191
06-14-65	21	661	100
07-05-65	21	702	83
07-26-65	21		
08-16-65	21		
09-06-65	21		
09-27-65	21		

clearing, respectively.

Table 20 gives ET values determined from reach 1 during the season of high potential ET in June and July for the before- and after-clearing period of the study. Included in the table are the sampling error (E_{ET_s}) and total measurement error (E_{ET}) for each ET value. The total bias error is nearly constant at $E_{ET_b}=\pm 46$ acre-ft (± 0.057 hm^3) per 14 days and has been omitted from the table. The ET values for some budget periods are obviously unreasonable and generally correspond with a large measurement error. Thus, those values with a total measurement error of $E_{ET}>\pm 550$ acre-ft (± 0.678 hm^3) are not included in the subsequent computations. The criteria used in selecting this error limitation was arbitrarily established such that no data were accepted in which the error exceeded the average June and July potential ET of approximately 550 acre-ft (0.678 hm^3) per 14 days. A plot of the ET values in table 20 with measurement errors less than ± 550 acre-ft (± 0.678 hm^3) is shown in figure 16.

These ET data define average 14-day rates of $\overline{ET}_B=320$ acre-ft (0.395 hm^3) before clearing and $\overline{ET}_A=181$ acre-ft (0.223 hm^3) after clearing. The average change (reduction) in ET as a result of clearing is $\Delta\overline{ET}=320-181=139$ acre-ft (0.171 hm^3) per 14 days for the June–July period in reach 1.

The standard deviation of the 12 before-clearing ET values in table 20 is $s_{ET_B}=\pm 79$ acre-ft (± 0.097 hm^3) or $\pm$ 25 percent of $\overline{ET}_B$. The standard deviation of the 19 after-clearing ET values is $s_{ET_A}=\pm 77$ acre-ft (± 0.095 hm^3) or ± 43 percent of $\overline{ET}_A$. The near-equal values of s_{ET_B} and s_{ET_A} indicate that the variability in the computed values of ET is a function of the total volume of water passing through the reach—which does

TABLE 20.—*ET values obtained before and after clearing and their corresponding sampling, and total measurement errors for selected 14-day budget periods during June and July, reach 1*

[All values in acre-ft per 14 days]

Before clearing				After clearing			
Day[1]	ET_B	E_{ET_s}	E_{ET_B}	Day[1]	ET_A	E_{ET_s}	E_{ET_A}
6-11-63	389	±172	±178	6-12-67[2]	218	±140	±151
6-25-63	270	151	158	7-3-67[2]	133	143	154
7-9-63	306	153	159	7-24-67[2,3]	275	857	859
7-23-63	254	159	165	6-24-68[2]	269	261	267
8-06-63[3]	446	596	598	7-8-68	351	205	211
6-15-64[2]	248	153	163	7-22-68	25	167	173
7-6-64[2]	283	141	152	8-5-68	254	424	426
7-27-64[2,3]	704	671	673	6-9-69	138	178	184
6-7-65[2]	271	161	171	6-23-69	192	170	176
6-28-65[2]	299	138	149	7-7-69	228	170	176
7-19-65[2]	313	142	153	7-21-69	183	256	261
6-20-66[2]	447	192	200	8-4-69	89	173	183
7-11-66[2]	492	160	170	6-15-70	136	163	170
8-1-66[2]	270	352	357	6-29-70	269	160	166
				7-13-70	154	153	164
				7-27-70	209	170	176
				6-14-71	93	159	165
				6-28-71	106	157	163
				7-12-71	203	158	165
				7-26-71	192	171	177
$k=12$ periods				$k=19$ periods			
$\overline{ET}_B=320$				$\overline{ET}_A=181$			
$s_{\overline{ET}_B}=\pm 79$				$s_{\overline{ET}_A}=\pm 77$			
$E_{\overline{ET}_s}=\pm 182$				$E_{\overline{ET}_s}=\pm 199$			
$E_{\overline{ET}_B}=\pm 189$				$E_{\overline{ET}_A}=\pm 205$			

[1]Day refers to last day of 14-day budget period.
[2]ET and error values originally computed for 21-day budget period but adjusted to 14-day budget period.
[3]ET and error values excluded from computations because $E_{ET}>\pm 550$ acre-ft.

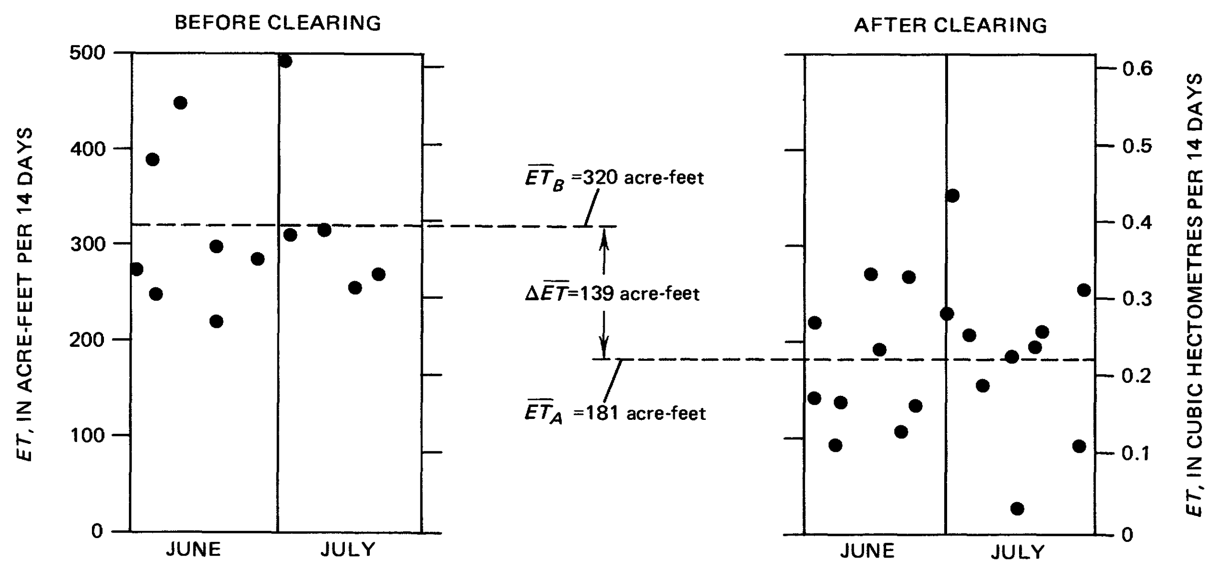

FIGURE 16.—Water-budget ET values in reach 1 for the before-and after-clearing periods in June and July and the average change in ET as a result of phreatophyte clearing. Values are plotted on the day corresponding to the middle of the budget period.

not change appreciably with time—and not the magnitude of ET.

The average measurement errors associated with $\overline{ET}_B$ and $\overline{ET}_A$ may be computed from

$$E_{\overline{ET}}=\frac{1}{k}\Big(\sum_{t=1}^{k} E^2_{ET_t}\Big)^{1/2}, \quad (41)$$

where $E_{\overline{ET}}$ is the average total measurement error of ET for k budget periods and E_{ET_t} is the total measurement error in ET for budget period t. Applying equation 41 using the total measurement errors E_{ET_B} and E_{ET_A} in table 20 gives $E_{\overline{ET}_B}=\pm 189$ acre-ft (± 0.233 hm^3) and $E_{\overline{ET}_A}=\pm 205$ acre-ft (± 0.253 hm^3).

Assuming independence between $\overline{ET}_B$ and $\overline{ET}_A$, the standard deviation of $\Delta\overline{ET}$ is $s_{\Delta\overline{ET}}=\sqrt{s^2\overline{ET}_B+s^2\overline{ET}_A}=\sqrt{79^2+77^2}=\pm 110$ acre-ft (± 0.136 hm^3) or ± 79 percent of $\Delta\overline{ET}$. Much of this variability is real, reflecting year to year variations in the potential ET rate and in the moisture available for evaporation and transpiration. The remaining variation results from measurement errors in the ET components.

The average measurement error for $\Delta\overline{ET}$ includes only the sampling errors ($E_{\overline{ET}_s}$) computed from equation 37; the bias errors ($E_{\overline{ET}_b}$) are omitted because they are one directional and essentially constant for all estimates of ET and thus cancel in the error computation. The average sampling errors ($E_{\overline{ET}_s}$) of ± 182 acre-ft (± 0.224 hm^3) before clearing and ± 199 acre-ft (± 0.245h m^3) after clearing were obtained by substituting the corresponding E_{ET_s} error values of table 20 in equation 41. The average measurement error in $\Delta\overline{ET}$ is then $E_{\Delta\overline{ET}}=\sqrt{182^2+199^2}=\pm 270$ acre-ft (± 0.333 hm^3), which is nearly $\pm$ 200 percent of $\Delta\overline{ET}$ and exceeds the standard deviation of $\Delta\overline{ET}$ by 2½ times.

The fact that these measurement errors in ET and ΔET are significantly greater than their standard deviations indicates that the assumptions and criteria used to obtain the measurement errors produce an overestimate of the true measurement variability in ET. These total measurement errors must therefore be considered only an indicator of the relative significance of each ET value.

No evaluation was made of the winter ET rates in this report; however, Hanson, Kipple, and Culler (1972, fig. 4) showed that the winter rates average substantially lower than the summer rates before clearing and that no significant change in the winter rates can be detected after clearing. The measurement errors in ET are also generally higher during the winter than during the spring and early summer months as indicated in figure 15*A*. Estimates of ET for the winter months of typically low rates are therefore less reliable than the summer estimates.

DISCUSSION OF RESULTS

Of the 12 components of the water budget, the Gila River inflow (Q_I) and outflow (Q_O) are generally the most significant, reaching maximum rates during the winter and spring snowmelt period and during the late summer thunderstorm period. Tributary inflow (Q_T) occured only 4 percent of the time during the nine-year study period, and even though some of these events did produce large volumes of inflow, this component is considered to be one of the least significant during the study period. Of the more important components in the water budget, moisture-content changes—ΔM_S, ΔM_I, ΔM_C, and ΔM_{TC}—are the most difficult to measure. Moisture change, particularly in the capillary zone, generally becomes significant during periods of low streamflow. Except for basin-fill inflow, which was assumed constant, the ground-water inflow (G_I) and outflow (G_O) are the least variable components during the year, fluctuating only in response to seasonal changes in the downvalley ground-water slope. The only component of any consequence in the water budget that was not measured in this study is depression storage—that water which fills side channels and depressions in the flood plain during overbank flooding. The relatively infrequent occurrence of depression storage and the difficulties in measuring this component did not justify including it in the water-budget analysis.

The total measurement error for most of the water-budget components consists primarily of a sampling error which is dependent on the number of observation points used to evaluate the component. The sampling error is time variant—reflecting both the variability in repetitive measurements and the error due to missing data. Included in the total measurement error is a bias error which reflects a consistent overestimate or underestimate of the water-budget component. Only the basin-fill inflow and the ground-water inflow and outflow components introduce a measurable bias in the computation of ET. Because of the uncertainty in the estimate of the basin-fill inflow, the bias of this component is assumed to equal the total basin-fill inflow. The bias in the ground-water inflow and outflow components reflect possible errors in the determination of the average transmissivity for the study area and inaccurate measurements of the width of saturated alluvium at the inflow and outflow cross sections of each reach.

The magnitude of the measurement error of ET is directly related to the total volume of water moving through the reach and not the magnitude of ET. Thus, ET computed from a budget period of high streamflow has a correspondingly large measurement error. Fortunately, high streamflow is generally limited to the winter months, when ET is minimal and a few weeks in late summer when runoff from thunderstorms occurs. During the midsummer months of maximum ET, the measurement errors become minimal because of low streamflow and negligible tributary inflow and precipitation.

The measurement errors of ET for the summer periods investigated in this report (table 20) are ±59 percent of the computed average before-clearing value of ET and ±113 percent of the computed average after-clearing ET rate. The measurement error of the average change in ET as a result of clearing is nearly ±200 percent of the computed change for these summer periods. The measurement errors of ET and change in ET for the winter periods are generally even greater than for the summer periods.

The large measurement errors computed in this study would make it appear that the ET rates derived from the water budget do not provide reliable estimates of the true ET rates. Most of these computed errors can be assumed, however, to exceed the actual measurement errors of ET and ΔET because the criteria used to estimate the error of each component give values that would be expected to exceed their standard error. This is substantiated by a comparison of the $E_{\overline{ET}}$ values with the significantly lower $s_{\overline{ET}}$ values in table 20. Because $s_{\overline{ET}}$ includes both the true measurement errors in the data and real variations reflecting year to year differences in moisture available for evapotranspiration, it is apparent that the computed measurement errors are too high. These data show, in fact, that reliable estimates of ET for the summer periods can be obtained and that a significant difference in ET could be detected as a result of clearing the phreatophytes from the flood plain.

Even though most of the computed measurement errors for ET probably exceed the actual measurement errors, they do provide a good indication of the *relative* significance of each ET value. These measurement errors were used as the basis for selecting the most reliable ET estimate in evaluating the average before clearing and after clearing ET rates from all reaches in the study. A discussion of the application of these measurement errors to the evaluation of the average ET rates will be included in a subsequent paper in this series.

Studies have been carried out to evaluate the variability in the ET data due to differences in moisture available for vaporization and differences in the potential to remove the available water. In addition, the differences in ET between reaches due to differences in vegetative cover has been evaluated. The results of these studies will also be included in a subsequent report.

DEVELOPMENT OF EQUATIONS DESCRIBING UNADJUSTED SAMPLING ERROR, ∇

An estimate of the average value of a hydrologic variable for a given area such as precipitation approaches the population mean ($\overline{P}$) as the number of sample points used for the estimation increases. Measures of most hydrologic variables are sample realizations of the time series in which they occur and therefore are frequently autocorrelated. Thus, the rate at which any estimate of a given variable approaches its mean value is unknown.

If n sample points are used to estimate the mean $\overline{P}_n$, the standard deviation of the departure between $\overline{P}_n$ and $\overline{P}_m$ (where $m<n$) decreases as m approaches n in a manner indicated by curve $\overline{S}_m$ of figure 17. In this report $\overline{S}_m$ is referred to as the average missing-data error. A sampling error, ∇, exists at $m=n$, and a relation between sample size and total error which includes both $\overline{S}_m$ and ∇ can be described by curve $E_{\overline{P}_m}$ in figure 17. An approximation to the shape of curve $E_{\overline{P}_m}$ can be found by estimating ∇ such that

$$E^2{}_{\overline{P}_m}=\overline{S}^2{}_m+\nabla^2, \qquad (26)$$

where $E^2{}_{\overline{P}_m}$ is the variance of the departure between $\overline{P}_m$ and an estimate of the population mean, and is inversely proportional to the sample size m.

The procedure for determining $\overline{S}_m$ for all values of m has been described previously in this report. The purpose of this section is to describe the development of the equations used in estimating the sampling error, ∇.

An approximation of ∇ may be obtained from an evaluation of the rate of change in $\overline{S}_m$ as m approaches n. If the variance $\overline{S}^2{}_m$ is defined from m samples and the

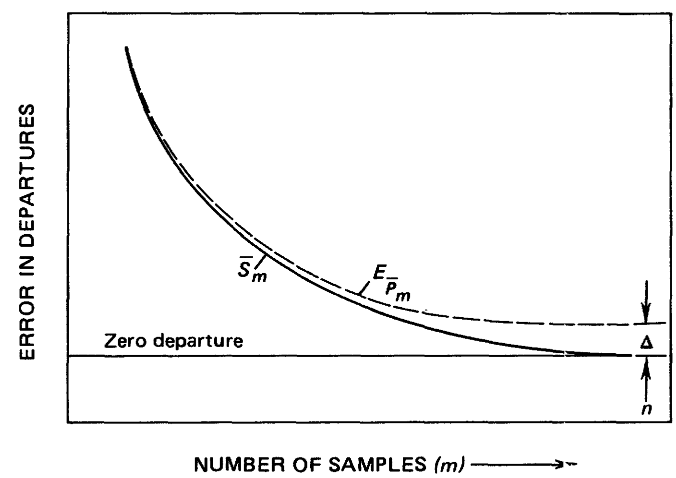

FIGURE 17.—General relation for error in departures of average of m samples from average of n samples (curve S_m), and error in departure of average of m samples from estimate of population mean (curve $E_{\overline{P}_m}$).

variance $\bar{S}^2{}_{2m}$ is defined from $2m$ samples, then the ratio $\frac{\bar{S}^2_m+\nabla^2}{\bar{S}^2_{2m}+\nabla^2}$ is equal to 2 with some residual error, $\xi_{m,2m}$. It can be assumed that the average rate of change in the square of this residual error will approach zero as m increases—or expressed in equation form,

$$\frac{\partial}{\partial\nabla}\xi^2_{1,2}+\frac{\partial}{\partial\nabla}\xi^2_{2,4}+\frac{\partial}{\partial\nabla}\xi^2_{4,8}+\cdots+\frac{\partial}{\partial\nabla}\xi^2_{n'/2,n'}=0, \quad (42)$$

where

$$\xi_{1,2}=2-\frac{\bar{S}^2_1+\nabla^2}{\bar{S}^2_2+\nabla^2}, \quad (43)$$

$$\xi_{2,4}=2-\frac{\bar{S}^2_2+\nabla^2}{\bar{S}^2_4+\nabla^2}, \quad (44)$$

$$\xi_{4,8}=2-\frac{\bar{S}^2_4+\nabla^2}{\bar{S}^2_8+\nabla^2}, \quad (45)$$

$$\xi_{n'/2,n'}=2-\frac{\bar{S}^2_{n'/2}+\nabla^2}{\bar{S}^2_{n'}+\nabla^2}, \quad (46)$$

and

n' is the largest even value less than or equal to n.

Solving the partial differential equation

$$\frac{\partial}{\partial\nabla}\xi^2_{1,2}=\frac{\partial}{\partial\nabla}\left(2-\frac{\bar{S}^2_1+\nabla^2}{\bar{S}^2_2+\nabla^2}\right) \text{ gives}$$

$$\nabla^4(\bar{S}^2_1-\bar{S}^2_2)+\nabla^2\left(4\bar{S}^2_1\bar{S}^2_2-3\bar{S}^2_2-\bar{S}^4_1\right)$$

$$+\left(3\bar{S}^2_1\bar{S}^2_2-2\bar{S}^6_2-\bar{S}^4_1\bar{S}^2_2\right)=0. \quad (47)$$

By replacing each $\frac{\partial}{\partial\nabla}\xi^2$ term in equation 42 with a general expression for equation 47 gives the previously shown quadratic equation

$$a\nabla^4+b\nabla^2+c=0, \quad (22)$$

where

$$a=\sum_{m=1}^{n'/2}\left(\bar{S}^2_m-\bar{S}^2_{2m}\right), \quad (23)$$

$$b=\sum_{m=1}^{n'/2}\left(4\bar{S}^2_m\bar{S}^2_{2m}-3\bar{S}^2_{2m}-\bar{S}^4_m\right), \quad (24)$$

and

$$c=\sum_{m=1}^{n'/2}\left(3\bar{S}^2_m\bar{S}^2_{2m}-2\bar{S}^6_{2m}-\bar{S}^4_m\bar{S}^2_{2m}\right). \quad (25)$$

The positive root of ∇ in equation 22 may then be obtained from

$$\nabla^2=\frac{-b\pm\sqrt{b^2-4ac}}{2a}. \quad (48)$$

Solving for ∇ in equation 48 using the coefficients a, b, and c obtained from equations 23–25 gives the least squares best-fit curve for ∇.

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